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THE UNIVERSITY OF ALBERTA

PHILOSOPHICAL IMPLICATIONS OF BELL'S THEOREM

BY

NIALL SHANKS

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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Date: May 22, 1987

Dedicated to the memory of my mother

JANET SHANKS

(1925 - 1985)

ABSTRACT

This study concerns Bell's Theorem that there can be no Bell local hidden variables theory for the quantum spin correlation statistics generated by pairs of spacelike separated spin - 1/2 particles in the singlet spin state. Since Bell's Theorem rests on two assumptions: hidden variables and Bell locality, Bell's Theorem leaves us with a dilemma. According to Bell's dilemma we are faced with a choice between the hidden variables assumption and the assumption of Bell locality. Most theorists accept Bell locality and call the hidden variables assumption into question.

After I have presented the general concept of a hidden variables theory and a variety of hidden variables strategies for quantum measurement statistics, I will present Bell's Theorem from both a theoretical and an experimental perspective.

This study will then deal with three questions:

1) Is Bell's Theorem an inevitable consequence of the use of classical probability theory in the analysis of quantum spin correlation measurement statistics?

2) What is the relevance of Bell's Theorem to the realist/anti-realist debate?

3) Is the standard view that quantum mechanics itself has no commitment to hidden variables correct?

In discussing these questions, this study aims to shift the focus of the debate concerning Bell's Theorem away from the hidden variables assumption and onto the Bell locality assumption.

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CHAPTER ONE

INTRODUCTION

> I don't like it and I'm sorry
> I ever had anything to do with it.
>
> E. Schroedinger[1]

SECTION ONE: THE HIDDEN VARIABLES QUESTION

The central concern of this study is with the philosophical implications of Bell's Theorem. Before turning to Bell's Theorem, however, it is important to discuss some of the issues which lie behind Bell's work.

Quantum mechanics makes essential use of probability in its description of the world. For more than sixty years a controversy has smouldered concerning the character of this probabilistic aspect of quantum mechanics and whether or not quantum mechanics is ultimately interpretable as a classical theory. For example, is there some level at which quantum reality may be considered both determinate and deterministic? One recalls Einstein's remark that God does not play dice, and certainly the drive to restore determinism to the physics of the microcosm has been one important strand in the history of quantum mechanics.

Let us dub those who believe that quantum mechanics is true ultimately of a determinate world hidden variables theorists. As noted above, some hidden variables theorists have been motivated by a desire to restore determinism to the physics of the microcosm.[2] Some theorists, however, have been motivated by a desire to retain the classical ontology of particles qua billiard balls writ small, but have gone on to view quantum reality as behaving in an inherently stochastic manner. The view that the denizens of the microcosm are determinate and yet behave indeterministically can be found in both the statistical[3] and stochastic[4] interpretations of quantum mechanics. Hidden variables theorists, in whatever form they appear, have constituted a small but vocal minority in the history of quantum mechanics.

Ranged against the hidden variables approaches to the interpretation of quantum mechanics is a variety of other interpretations. Dominant among these is the Copenhagen (or 'orthodox') Interpretation.[5] From a historical point of view it may be best to analyze the Copenhagen Interpretation as a 'soup' of related interpretations--and not necessarily a consistent soup at that.[6]

In Bohrian versions of the Copenhagen Interpretation it is claimed that the proper concern of quantum mechanics is with measurement results and their probabilities alone. Thus, when we measure an observable--say momentum--on a system, we can only properly speak of having performed a momentum-value determining experiment. As Jammer has commented:

> In fact it would be difficult to find a textbook of that period (1930-1950) which denied that 'the numerical value of a physical quantity has no meaning whatsoever until an observation upon it is performed.'[7]

On the Bohrian view,[8] we cannot, at the time we perform a momentum-value determining experiment, even ask about what happens to the position observable on a system--the rationale being that no experiment can simultaneously determine exact values for position and momentum.[9]

In Copenhagen lore, a position-value determining experiment requires a configuration of the apparatus incompatible with that required by a momentum-value determining experiment. So what happens to position when we determine momentum? Well, don't ask! Indeed, on the Bohrian view we cannot even speak of a microsystem in the context of any observable-value determining experiment, since according to that view, at the time of measurement both the measured system and the measuring apparatus form a conceptually indivisible totality concerning which nothing can be said of the parts short of changing the type of experiment being performed. This was the "tranquillizing philosophy" of quantum mechanics which Einstein decried.[10] This was also what Arthur Fine had in mind when he commented of quantum mechanics:

> It is rather the blackest of black box theories; a marvellous predictor but an incompetent explainer.[11]

The predictive success of quantum mechanics is well-documented --just consult any standard teaching text. The poverty of quantum mechanical explanation is usually equally well documented (unintentionally) in the same texts. Of course, Bohr's philosophy of quantum mechanics is anti-explanatory from the start. However, some orthodox theorists have tried to offer explanations of quantum phenomena. Even if those explanations are mere heuristic devices to serve pedagogical ends, it is nevertheless worth noting that they are very often unsatisfactory.

For instance, consider the so-called 'collapse of the wave-packet'. This is the process that putatively occurs when the state of a system relative to some observable--say position--changes instantaneously and indeterministically from a linear superposition of eigenstates of that observable (these states look like 'wave-states') to an eigenstate of that observable (when the system takes a definite value for position). One way in which this process has been explained is in terms of the doctrine of wave-particle duality. According to this doctrine, systems in the domain of quantum mechanics are ontological chameleons--sometimes they are like waves and sometimes they are like particles. The collapse of the wave-packet on this view represents the transition of a quantum system from its wavelike character to its particlelike character. (When such a system is in an eigenstate of some observable--say momentum--which does not commute with position, then the system will be spread out along the continuum of its position eigenvalues. According to the usual story, at the time of measurement this wavelike system collapses to a point-particle. The collapse is instantaneous--and this seems to involve some form of action-at-a-distance).

Sometimes at this point a sleight-of-hand is indulged in: we are told that the quantum state represents only our knowledge of the system. At the time of measurement, when we find the 'particle' at some spacetime location, _we_ undergo a transition from 'ignorance' to 'knowledge' of the particle's location. There is no action-at-a-distance here. The trouble is that there is sometimes a tendency to think of these waves as being diffracted--for instance in the two-slit experiment.[12] Now physical waves can be diffracted and can give rise to interference phenomena, but 'waves of knowledge' cannot.

Similar remarks can be made about the view that the wave function of quantum mechanics is not a physical wave but a 'wave of probability'. For instance, Heisenberg[13] suggests at one point that an electron can be considered to be a plane De Broglie wave subject to diffraction by holes in walls. Yet in the same discussion, he gives the wave function an epistemological interpretation when he comments:

> As our knowledge of the system does change discontinuously at each observation its mathematical representation must also change discontinuously; this is to be found in classical statistical theories as well as in the present theory.[14]

Another example of this interpretative equivocation is to be found in Merzbacher's text where he comments on the one hand that:

> . . . it is impossible that ψ be positive (or zero) everywhere, if destructive interference of the ψ waves is to account for the observed dark interference fringes . . .[15]

And on the other hand that:

> The conclusion is almost inevitable that ψ describes the behaviour of single particles, but that it has an intrinsic probabilistic meaning. The quantity $|\psi|^2$ would appear to measure the chance of finding the particle at a certain place.[16]

Some theorists have gone further still and have tried to marry the concepts of 'wave' and 'particle' together into the concept of 'wavicle'. Thus Bohm comments:

> The preceding discussion leads to the idea that an electron is neither a particle nor a wave, but is instead a third kind of object which has some, but not all of the properties of both waves and particles.[17]

Bohm soon abandoned this view, but what is important here is that the fundamental property of waves is their spatial extension whereas the fundamental property of a point-mass is its lack of spatial extension. It is hard to see, on the face of it, how a system could share these fundamental properties.

Putting the collapse of the wave-packet the other way round, so to speak, it is certainly little comfort to be told that the result of preparing a system into an eigenstate of momentum is that the system no longer takes a value for position, (not that it is spread out over the continuum of its position eigenvalues but that it literally takes no value for position). This is the view we get when we drop the 'wave-talk' altogether. According to the so-called 'eigenstate- eigenvalue link',[18] observables ascribed to systems by quantum mechanics only take values when the system is in an eigenstate of those observables. Since no momentum eigenstate is an eigenstate of position, the result of a momentum-eigenstate preparation is that the quantum system disappears, so to speak, leaving only its momentum behind. On this view a measurement of position may reveal a position-value for the system-though *what* it is whose position is being determined is left unclear since, by the present hypothesis, immediately before measurement the system is not located anywhere (or even spread out over physical space). Once again, incoherence threatens.

Suppose then that Fine is right and that quantum mechanics is an incompetent explainer of phenomena in its domain. The question now arises as to what a competent explanation of quantum phenomena would look like. Hidden variables interpretations of quantum mechanics aim to fill the explanatory gap by interpreting quantum mechanics as a theory true of classical objects. The offering of explanations of phenomena in terms of the unobserved is a realist enterprise (though not exclusively so), and realists, who regard themselves as committed to the existence of entities they employ in their explanations of phenomena, have often seen the need for a hidden variables interpretation of quantum mechanics. In particular, the classical corpuscular ontology of 'Dalton's buckshot' is coherent and well-understood: just the sort of thing to which one would wish to be committed! So traditionally it is realism, which takes the explanatory function of science seriously, that has provided the philosophical motives for seeking out some hidden variables interpretation of quantum mechanics.

The time has come to be more precise about what a hidden variables interpretation of quantum mechanics is. The best way to deal with this issue is to discuss generally what a hidden variables theory is, and then to discuss the special sense of 'hidden variables theory' that is relevant to the hidden variables issue in quantum mechanics.

In one good sense of the term, a hidden variables theory T' of phenomena in the domain of a theory T, is a theory which supplements the theoretical apparatus of T with new variables (i.e., physical quantities or observables) in order to correct some perceived defect or inadequacy of T.

An example of a hidden variables theory in this sense would be the FitzGerald-Lorentz theory of Fresnel aether theory. The perceived inadequacy of Fresnel aether theory was its failure to predict the null result of the Michelson-Morley experiment.[19]

According to Fresnel aether theory, the earth moved through an all-pervasive luminiferous aether (which was the medium through which light propagated). The Michelson-Morley experiment was designed to measure the motion of the earth through the aether. The experiment was intended to permit a comparison of the velocity of light relative to the earth with the velocity of light relative to the aether. The apparatus used in the experiment was designed to be able to detect differences in the velocity of light along several different spatial orientations of the apparatus. No such differences were detected. Did this imply, as Einstein was later to claim in 1905, that there was no aether? FitzGerald in 1892, and Lorentz independently, suggested that the null result of the experiment was indirectly caused by motion through the aether. This motion caused the apparatus to contract in the direction of motion through the aether with the contractions precisely specified by the famous FitzGerald-Lorentz contraction equation. The effect of relative contraction of the apparatus in the direction of motion was supposed to compensate for the real differences in the velocity of light along the various orientations of the apparatus and thus render these differences unobservable.[20] The hidden variables which save Fresnel aether theory are the contractions of the measuring apparatus.

A classic precedent[21] for the hidden variables debate in quantum mechanics, and one that raises questions for the issue of the reduction of one physical theory to another, was the debate between proponents (e.g., Boltzmann) and opponents (e.g., Mach) of the mechanistic interpretation of phenomenological thermodynamics in terms of classical mechanics. In this case the hidden variables were the multitude of small particles and their properties (initially unobserved) whose behaviour was postulated to explain thermodynamical phenomena.

Phenomenological thermodynamics was designed to give an account of the gross behaviour of macroscopic systems (eg. refrigerators and heat engines) in conditions corresponding to the specification of a limited number of thermodynamical variables such as pressure, volume, temperature, energy and entropy. While the equations of phenomenological thermodynamics were deterministic, the states of the theory did not determine unique descriptions of physical (as opposed to ideal) systems in the domain of the theory. Typically there will be a multitude of configurations of a physical system compatible with a given thermodynamical state. Consider a sample of gas in a bottle. Standardly there will be many ways to arrange the molecules of the gas that are compatible with a specification of the thermodynamical state of the gas. Thus one cannot infer the precise mechanical state nor the precise behaviour of a system in the real (as opposed to ideal) domain of phenomenological thermodynamics from its thermodynamical state and application of thermodynamical equations of motion.

Tolman describes the aim of the classical mechanical treatment of thermodynamics as follows:

> The desired explanation of thermodynamics depends on showing that the science of statistical mechanics provides an appropriate interpretation for such non-mechanical variables as temperature and entropy, and provides predictions as to the average behaviour of systems of many degrees of freedom in substantial agreement with the predictions of thermodynamics.[22]

The systems of many degrees of freedom to which Tolman refers would be systems composed of many classical mechanical particles. If one knew the classical mechanical state of such a system one could (at least theoretically) use the equations of motion of classical mechanics to precisely predict or retrodict the behaviour of the system. By the lights of classical mechanics, phenomenological thermodynamical states leave us in ignorance of the precise mechanical states of physical systems in the domain of the theory. A phenomenological thermodynamical state provides values for the thermodynamical quantities--volume, pressure, temperature, etc.--and so is determinate (in the sense of providing values for all of these quantities). By the lights of classical mechanics, such states are not complete (in the sense that they do not provide values for all of the physically relevant quantities--as judged by the lights of classical mechanics). Classical mechanics supplements the theoretical apparatus of phenomenological thermodynamics with variables such as the positions and momenta of the microcomponents of systems in the domain of phenomenological thermodynamics. This supplementation allows for a mechanical interpretation of the thermodynamical variables and for a restoration of determinism, in a sense to be explained, to this area of physics.

As noted previously, the restoration of determinism to the physics of the microcosm has, from a historical point of view, also been of importance to some hidden variables theorists in the area of quantum mechanics.

Classical or Laplacean determinism has two components:

(a) deterministic equations of motion for particles, and

(b) determinate and complete states.

Equations of motion specify how system states change over time. A system state is determinate relative to a theory T if it provides values for all the observables or variables in T. A system state is complete if it provides values for all physically significant quantities. Laplace believed that the states of classical mechanics were determinate and complete. Thus his demon, who knew the positions and momenta of all the particles in the universe, would be able to use the deterministic equations of motion of classical mechanics to precisely predict the future and to precisely retrodict the past.

A system in the domain of some theory T can fail to be Laplace-deterministic either because it obeys a stochastic equation of motion or because its states are not determinate and complete. Indeed, both of the above may obtain. For instance, the equations of phenomenological thermodynamics were deterministic but the states of the theory, while they were determinate (they provided values for the thermodynamical variables), were not complete in the sense of providing values for all the physically significant variables. 'Determinacy of state' is a theory-relative formal concept. 'Completeness of state' is a concept derived from considerations of descriptive adequacy of theory. By the lights of classical mechanics, phenomenological thermodynamics was not descriptively adequate. The reduction of phenomenological thermodynamics to classical mechanics shows that those states of phenomenological thermodynamics, while they were incomplete, were nonetheless completable by the specification of the positions and momenta of the microsystems making up systems in the domain of phenomenological thermodynamics. The specification of determinate and complete states for such systems allows these systems to be treated in a Laplacedeterministic fashion.

The procedure is common in the history of science: when faced with indeterminism or randomness theorists standardly try to eliminate the indeterminism by a more complete specification of the systems concerned which takes into account all (or more) of the factors involved. This is true in fields as widely separated as physics and economics -- indeed, in just about any area of human endeavour where use is made of probabilistic or statistical methods. Of course, it is possible that even when all the factors have been taken into account, a system may really behave indeterministically (by obeying a stochastic equation of motion). It is nevertheless true that indeterministic behaviour of systems is often seen as calling for a more complete description of those systems.

This brings the discussion at last to quantum mechanics. If quantum mechanics is a theory true of classical particles--'billiard balls writ small'--then it is also true that quantum states are not determinate let alone complete. That is, no quantum state determines simultaneous exact values for all quantum mechanical observables ascribed to a system in the domain of the theory. The hidden variables question in quantum mechanics is this: is there some way to interpolate values for all quantum mechanical observables and possibly for some new observables as well, for all quantum states at all times? If this is possible then quantum mechanics will be said to be classically completable. Even if quantum mechanics were classically completable, it would not follow from this that there is a Laplace-deterministic theory of the microcosm--that would require that the new determinate and complete states be subject to deterministic equations of motion.

A theory whose states are determinate and complete will be called a classical theory. Classical theories may be either deterministic or stochastic, depending on the character of their equations of motion. The hidden variables issue in quantum mechanics is the issue of whether quantum mechanics is a classical theory. The hidden variables issue, then, primarily concerns the character of the states of systems in the domain of quantum mechanics. This issue can be pursued independently of considerations of time-development of state. Necessary for quantum mechanics to be either classical and deterministic or classical and stochastic is that its states be determinate and complete or that it be possible to interpret quantum mechanics in terms of a classical theory whose states are determinate and complete.

An important strand of the hidden variables enterprise in quantum mechanics has been determinacy. Is it possible, for any quantum state at any time, to interpolate simultaneous exact values for all quantum mechanical observables on a system. A determinate state at time t need be nothing more than a list of those observable values.

Let $O_1, O_2, \ldots, O_n$ be the observables of a given theory T. If $O_1, O_2, \ldots, O_n$ completely describe systems in the domain of T then a complete and determinate state of T will be a list $\lambda = (o_1, o_2, \ldots, o_n)$ of values of those observables. Suppose that T does not completely describe systems in its domain. Suppose that T' (a hidden variables theory with respect to T) does completely describe those same systems. There are at least two possibilities here. First, it may be the case that the observables of T are all of ontological importance (in the sense of characterizing the nature of physical systems). In this case the observables of T' will include those of T as a proper subset. The observables in T' will be $0_1, 0_2, \ldots 0_n, \ldots, 0_p$. In this case a state that is determinate and complete will be a list $\lambda^* = (o_1, o_2, \ldots o_n, \ldots, o_p)$ of the values of those observables. The list λ^* contains the list λ as a sub-list. With respect to systems in the common domain of T and T', the state λ will be determinate with respect to T but not complete. If T were Fresnel aether theory then T' would be the FitzGerald-Lorentz contraction theory.[23]

A second possibility is this: it may be the case that some or all of the observables of T are not viewed as being ontologically significant relative to T'. In this case T' will replace the T-observables $0_1, 0_2, \ldots, 0_n$ by new observables $0_1^*, 0_2^*, \ldots 0_n^*, \ldots$ If T' completely describes systems in the domain of itself and T then a list $\lambda^{**} = (o_1^*, o_2^*, \ldots, o_n^*, \ldots)$ will be a determinate and complete state. In the case of the reduction of phenomenological thermodynamics to classical mechanics the non-mechanical thermodynamical observables are reinterpreted in terms of mechanical observables.

Any state λ which assigns simultaneous exact values to all observables in a given theory T will be called a classically determinate state with respect to T. Any state λ' which only assigns values to some observables in T will be classically indeterminate with respect to T. Quantum states, if they are states of particle-systems, are classically indeterminate. A classical mechanical state of a particle-system will be classically determinate with respect to classical mechanics. Thermodynamical states are classically determinate. In the reduction of T (= thermodynamics) to T' (= classical mechanics) the thermodynamical observables are not preserved. Thermodynamical states are classically determinate but not complete.

Systems fully described only by classically determinate states will be called classically determinate systems. The hidden variables question as it bears on quantum mechanics is a minimal one: is quantum mechanics a theory true of classically determinate systems--systems which take simultaneous exact values for all quantum mechanical observables? The reason why the hidden variables issue in quantum mechanics concerns this minimal question is largely because of the nature of extant 'no hidden variables' proofs which aim to show that quantum observables cannot all take simultaneous exact values, and hence that quantum mechanics cannot be interpreted as a theory true of classically determinate systems.

It will be seen later in this study that certain realists--those who believe quantum mechanics is true of measurement-independent classical particles (Dalton's buckshot)--have tried to interpret quantum mechanics in terms of states λ which are lists of measurement-independent observable values for all observables assigned to systems by quantum mechanics. If measurement does not disturb the system measured, or disturbs the system measured in a way that can be precisely calculated and allowed for, then measurement could reveal those pre-existing observable-values.

Some hidden variables theorists[24] regard the states λ as determining only measurement results. In particular, the λ may then be regarded as lists of observable-values which would be found were appropriate measurements performed. There is no inference from the fact that measurement results are classically determinate to the claim that systems take those values independently of measurement, or even that there are measurement-independent systems.

It is also important to differentiate between non-contextual and contextual hidden variables theories. Whether a hidden variables theory is contextual or not depends on how fine-grained classes of measurements are. Contextual hidden variables theories, unlike non-contextual theories, give a role to the type of measurement performed, even for a given observable. Briefly put, a quantum observable O_j may belong to more than one maximal commuting set of quantum observables. In contextual theories the value taken by O_j may change from context to context--from one in which O_j is measured along with one maximal commuting set of observables to one in which it is measured along with another such set. The distinction between contextual and non-contextual hidden variables theories is important and will be discussed further in the next chapter.

SECTION TWO: OUTLINE OF CHAPTERS

This study concerns Bell's Theorem. Bell's theorem[25] has constituted a focal point for investigations over the last two decades into the hidden variables question in quantum mechanics. Bell's theorem states that hidden variables, in the sense of classically determinate states , are inconsistent with the statistical predictions of quantum mechanics if quantum mechanics satisfies a specific 'no- action-at-a-distance' or 'locality' condition known as Bell locality. Bell's argument can be applied to systems which are experimentally accessible -- thus to some extent the hidden variables question is not a question of mere theoretical interest. Bell's dilemma is this: in the light of the statistical predictions of quantum mechanics, and to some extent in the light of the evidence, we must choose between analyzing quantum mechanics in terms of classically determinate states and in terms of the Bell locality condition. In the recent history of analyses of Bell's theorem, it has been commonplace to view quantum mechanics as being Bell local and thus as being inconsistent with hidden variables.

This study focusses on three questions:

Question One. Is Bell's Theorem an inevitable consequence of the use of classical probability theory in the analysis of quantum spin correlation measurement statistics?

Question Two. What are the implications of the Bell theorem for the realist/anti-realist debate?

Question Three. Is the standard view that quantum mechanics itself has no commitment to hidden variables correct?

In the remainder of this Chapter, such formal concepts from probability theory and quantum mechanics as will be useful in the rest of the study will be presented.

In Chapter Two, the hidden variable issue in quantum mechanics will be made precise. The 'no hidden variables' proofs of von Neuman and Kochen and Specker will be discussed. Two examples of hidden variables theories in the context of quantum mechanics will be presented.

In Chapter Three, the Einstein, Podolsky, Rosen paradox (EPR) will be presented and the Bell theorem will be discussed against the backdrop of EPR from both a theoretical and an experimental perspective.

In Chapter Four, Question One will be addressed with respect to the work of Arthur Fine, who views the Bell theorem as heralding the advent of some non-standard (yet 'classical') probability theory. These matters will be pursued in connection with his statistical random variable model, prism model and synchronization model proposals.

In Chapter Five, Question Two will be addressed. A variety of realist and anti-realist positions will be examined from the standpoint of Bell's theorem. Important assumptions underlying the Bell theorem and certain influential claims as to the philosophical significance of the Bell theorem will be subject to critical scrutiny.

In Chapter Six, Question Three will be pursued in connection with some recent proposals in the field of quantum measurement statistics. Traditional views about the relation of quantum mechanics to the Bell theorem will be challenged.

SECTION THREE: CLASSICAL PROBABILITY THEORY

In this section, classical probability theory is explicated with a view to introducing the notation and concepts which will appear elsewhere in the study.

A classical probability space (CPS) is to be understood as follows:

Definition 1. A CPS is a triple $\Omega = \langle \Lambda, \sigma(\Lambda), P \rangle$, where Λ is a non-empty set, $\sigma(\Lambda)$ is a σ-algebra of subsets of Λ (that is, it is a set of subsets of Λ which includes Λ and is closed under countable unions and complementations), and P is a function called probability defined on $\sigma(\Lambda)$.

If a CPS is being used to model an experiment then $\lambda \in \Lambda$ can be thought of as distinct outcomes (e.g., 'head', 'tail' in coin-tossing experiments.) Sets in $\sigma(\Lambda)$ are called events.

The probability function P has the following properties:

1) $P(\Lambda) = 1$

2) $P(A) \geq 0$, for $A \in \sigma(\Lambda)$

3) If $[A_i : i \in J]$, for J some appropriate index set, is a countable partition of an event A, then $P(A) = \sum_{i \in J} P(A_i)$.

Definition 2. A random variable is a real-valued measurable function $f: \Lambda \rightarrow R$, R the real line.

Definition 3. A random variable f is a measurable function if, for each point $t \in R$, $[\lambda : f(\lambda) \leq t]$ is an event in $\sigma(\Lambda)$.

The mapping f induces a transfer of probability from sets in $\sigma(\Lambda)$ to measurable (i.e., Borel) subsets of R. Let M be any Borel set in the set B_R of the Borel subsets of R. The probability $P_f(M)$ associated with this set under the map f is defined as follows:

Definition 4.

$P_f(M) =_{df} P(f^{-1}(M)) = P([\lambda : f(\lambda) \in M])$

Since $f^{-1}(R) = \Lambda$ and, (where $\bigcup$ indicates union of disjoint sets), $f^{-1}(\bigcup_i M_i) = \bigcup_i f^{-1}(M_i)$, it is the case that:

1) $P_f(R) = 1$,

2) $P_f(M) \geq 0$,

3) $P_f(\bigcup_{i=1}^{\infty} M_i) = \sum_{i=1}^{\infty} P_f(M_i)$

P_f is thus a probability measure on the set B_R of Borel subsets of the reals. This measure is called the probability measure induced by the random variable f.

Definition 5. The cumulative distribution function F_f associated with the random variable f is defined by the following expression:

$F_f(t) = P([\lambda : f(\lambda) \leq t])$, $t \in R$.

If the cumulative distribution function is continuous then a density function ρ may be defined in such a way that the probability associated with any $M \in B_R$, $(M = [a,b])$, is given by:

1) $P_f(M) = P([\lambda : f(\lambda) \in M]) = \int_a^b \rho(u)\, du$.

That the density function is normalized means:

2) $\int_{-\infty}^{\infty} \rho(u)\, du = 1$

The cumulative distribution function may be retrieved as:

3) $F_f(t) = \int_{-\infty}^{t} \rho(u)\, du$

with

4) $\rho(t) = \frac{d}{dt} F_f(t)$

Definition 6. The mathematical expectation E(f) of a random variable f is the probability-weighted average of the values of f:

$$E(f) = \int_{\Lambda} f(\lambda)\, dP = \int_{\Lambda} f(\lambda)\rho(\lambda)\, d\lambda$$

Let g(x) be a real-valued function of a single real variable, ($g: R_1 \rightarrow R_2$, R_1, R_2 images of the real line R).

Definition 7. A function g(x) is a Borel function iff for every Borel set M_2 of R_2, the inverse image $g^{-1}(M_2) = N_1$ is a Borel set in R_1. (n-place Borel functions are defined similarly: $g(1, \ldots, n) : R_1^n \rightarrow R_2^n$, where $R^n = R_1 \times \ldots \times R_n$).

A Borel function of a random variable will itself be a random variable, e.g., the random variable h = g(f). The two are related as follows:

1) $[\lambda : h(\lambda) \in M] = [\lambda : f(\lambda) \in g^{-1}(M)]$,

2) $P_h(M) = P_f(g^{-1}(M))$.

SECTION FOUR: PROBABILITY THEORY AND CLASSICAL PHYSICS

Some of the concepts introduced in Section Three may have their usefulness illustrated by a consideration of classical mechanics. In what follows, considerations of time-development of state will be ignored.

Two important elements of classical mechanics are the set S of system states $\emptyset$ and the set 0 of observables A (e.g., position, momentum, etc.). The system states of classical mechanics can be thought of as elements $\lambda \in \Lambda$. The non-empty set Λ of system states is called a phase space. The classical mechanical observables can be thought of as random variables defined on Λ.

The state of a classical mechanical system is given by a specification of the positions and momenta of the particles making up the system. An arbitrarily complex system[26] can be described by some set of n position values $q_1, q_2, \ldots, q_n$ and n momentum values $p_1, p_2, \ldots, p_n$, i.e., by 2n parameters. This number n of independent position values needed in order to describe the system is called the number of degrees of freedom. The position values $q_1, q_2, \ldots, q_n$ and momentum values $p_1, p_2, \ldots, p_n$ can be thought of as coordinates specifying some point in a phase space. Hence the state of the system is completely specified by giving the coordinates of position and momentum for the system. If the system consists of N point particles and each particle is described by three position coordinates then $n = 3N$. Any state of the system is given by a set of numbers $[q_1, \ldots, q_n, p_1, \ldots p_n]$ corresponding to a point in a phase space of 2n dimensions. This phase space is standardly treated as being isomorphic to a 2n dimensional Euclidean space.

Classical mechanics is classically determinate, and this finds its expression in the fact that each $\lambda \in \Lambda$ determines exact values for all classical mechanical observables (analyzed as random variables on Λ). This is in essence the random variable/phase space setting for classical mechanics.

The states λ of classical mechanics are called pure states because they provide maximal amounts of information about systems in the domain of the theory. If the classical mechanical state of a system is known then within the framework of classical mechanics there is nothing more to be known about the system.

In the more general theory of classical statistical mechanics, the concept of 'state' is extended to include not only pure states but also mixed states which provide incomplete information concerning physical systems.

In order to discuss the probabilistic element of classical physics we require a CPS. Let the CPS be $\Omega = \langle \Lambda, \sigma(\Lambda), P \rangle$, where is a non-empty set of classical mechanical pure states.

The probability measures $P^{\lambda}_{f_A}$ associated with each observable f_A and state λ($P^{\lambda}_{f_A}$ is the measure on R induced by the map $f_A : \Lambda \rightarrow R$), have the property of being dispersion-free.

Definition 1. A state λ is dispersion-free iff, for every random variable f_A (each representing a classical mechanical observable A), there exists a $t \in R$ such that $f_A(\lambda) = t$ and $P^{\lambda}_{f_A}(M) = 1$ iff $t \in M$ and 0 otherwise.

Definition 2. A probability measure $P^{\lambda}_{f_A}$ is dispersion-free iff it has all its weight concentrated on some point $t \in R$. Such measures are also called atomic measures.

The mixed states of classical statistical mechanics can be viewed as probability weighted sums of classical mechanical pure states. A mixed state can be represented by a density function $\rho(\lambda)$ as follows: let Γ be a subset of the space Λ of classical mechanical states. A mixed state ξ can then be written:

1) $\xi = \int_{\Lambda} \lambda \rho(\lambda) d\lambda = \int_{\Lambda} \lambda \, dP,$

for $\lambda \in \Gamma$ and where

2) $\int_{\Gamma} \rho(\lambda) d\lambda = 1.$

Each classical mechanical pure state is a trivial mixture of itself, so it must be the non-trivial mixed states which provide incomplete information concerning systems. The density function representation of mixed states is not suited to the expression of the idea that a pure state is a trivial mixture of itself -- since placing all the probability weight on a single point yields an infinite density. Luckily there is another way of proceeding:

Definition 3. A classical statistical mixed state ξ is a mixture of a set Γ of classical mechanical pure states λ ($\Gamma \subseteq \Lambda$) if there is a phase space measure Δ (non-negative, of total measure 1, defined on a σ-algebra of subsets of Γ) with respect to which $f_A(\lambda)$ is a measurable function of λ, for all random variables f_A and $\lambda \in \Lambda$, and such that:

(a) $P_{f_A}(M) = \Delta(f_A{}^{-1}(M))$

(b) $E(f_A)_{\xi} = \int_{\Gamma} f_A(\lambda) \, d\Delta$

The probability measures associated with non-trivial mixed states enjoy the property of dispersion. A dispersed measure on the real line R has its weight spread over an interval of the real line R. A dispersed phase space measure has its weight 'smeared' over a region of phase space.

In classical statistical mechanics, if all that is known of a system is that it is in some non-trivial mixed state , then one is ignorant of the precise mechanical pure state λ of the system. Finally, the property (b) of Definition 3 above illustrates what is meant by retrieving an expectation value for an observable in a given state as a phase space average.

Since the observables of classical statistical mechanics are associated with random variables on a common phase space, they will all have well-defined joint probability distributions. For n random variables, joint measures will be induced on the measurable subsets of $R^n = R_1 \times \ldots \times R_n$, with marginal probability distributions being induced on the various subspaces of R^n. More precisely:

Definition 4. A joint distribution for random variables $f_{A_1}, \ldots, f_{A_n}$ is a measure $P_{f_{A_1}, \ldots, f_{A_n}}$ on the measurable subsets of $R^n = R_1 \times \ldots \times R_n$ returning each measure associated with each random variable as a marginal probability, and thus as satisfying:

$$P_{f_{A_1}, \ldots, f_{A_i}, \ldots f_{A_n}} (R \times \ldots \times S \times \ldots \times R) = P_{f_{A_i}} (S),$$

where the Borel set S occurs in the i^{th} place in the cartesian product.[27]

SECTION FIVE: ELEMENTARY QUANTUM MECHANICS

Von Neumann's axiomatization of quantum mechanics proceeds essentially as follows:

1) There is a map $S:Q \longrightarrow H$ for any system in the domain of quantum mechanics to an associated Hilbert space H whose unit rays completely describe the states of the system.

2) There is a map $I:[O] \rightarrow [\mathbf{A}]$ from the set of quantum mechanical observables O to the set of self-adjoint operators **A** on the associated Hilbert space H.[28]

3) The Projection Postulate. If a measurement of an observable A yields the eigenvalue a_i then the state of the system immediately after measurement is the eigenstate $\O_i$ of A corresponding to the eigenvalue a_i (Ignoring degeneracy).

4) The Born Rule. If the quantum state of the system is:

$$\Psi = \Sigma_i c_i \O_i$$

where $[\O_i]$ is a complete orthonormal set of eigenstates of an observable A, with corresponding eigenvalues $[a_i]$, then $|c_i|^2$ gives the probability that the value of A will be found upon measurement to be a_i.

5) The Schroedinger Equation. The time-development of the state vector $\O$ is determined by the time-dependent Schroedinger equation:

$$\mathbf{H}\O = i\hbar\, \partial \O / \partial t,$$

where **H** is the Hamiltonian operator and $\hbar$ is Planck's constant divided by 2π.

A Hilbert space is a strictly positive inner product linear vector space (defined, in the case of quantum mechanics, over the field of complex numbers). The space H is separable and complete with respect to the metric generated by the inner product.

The strict positiveness of the inner product means:

1) $|\O|^2 = (\O,\O) > 0,$

unless $\O$ is the zero vector.

If $\O$ is a unit norm element of H then:

2) $(\O, \O) = 1,$

3) $(\O,\Psi) = 0$ implies that $\O$ and Ψ are orthogonal.

The vectors in a given set are said to be linearly independent if no member of the set can be written as a linear combination of other members of the set. The maximum number of linearly independent vectors in H is the dimensionality of H. Any set of linearly independent vectors in terms of which all other vectors in H may be expanded, forms a basis for H. If a set of linearly independent vectors forms a basis for H, and if the basis vectors are mutually orthogonal unit norm elements, then the basis is called an orthonormal basis. Furthermore, any subspace of a Hilbert space is itself a Hilbert space.

The observables A of quantum mechanics are associated with self-adjoint operators. In the present study the concern is with discrete observables. Furthermore, for the sake of simplicity, complications which arise out of degeneracy are ignored unless relevant.

The operators considered here are bounded linear transformations of H into H. Such operators have linear manifolds for both their domain and their range.[29]

The adjoint of any such operator $\mathbf{A}$ is $\mathbf{A}^{+}$ and is defined as follows:

1) $(\phi, \mathbf{A}\Psi) = (\mathbf{A}^{+}\phi, \Psi)$,

for all $\phi, \Psi \in H$. $\mathbf{A}$ is self-adjoint if $\mathbf{A}^{+}=\mathbf{A}$.

The eigenvalue equation for a self-adjoint operator is:

2) $\mathbf{A}\phi = a\phi$,

where a is a real number. For any self-adjoint operator, only a proper subset of states $\phi \in H$ can satisfy the corresponding eigenvalue equation. The vectors which satisfy this equation are called the eigenvectors of $\mathbf{A}$ (and they correspond to the eigenstates of the corresponding observable). The associated real numbers a are the eigenvalues corresponding to those eigenvectors. As a set, the normalized eigenvectors of any (non-degenerate) self-adjoint operator **A** will form a complete orthonormal basis for the Hilbert space H.

A subset of the set of self-adjoint operators is the set [**P**] of projection operators on H and the measurable subspaces of H. Projection operators have the property of idempotence. To wit, for any projection operator **P** and state vector $\emptyset \in H$

(1) $\mathbf{PP}\emptyset = \mathbf{P}\emptyset$.

A projection operator does its work by projecting any element $\emptyset$ of H onto its component which lies in the subspace corresponding to the operator.

Of special interest is the one-one correspondence between the normalized square-integrable solutions of the Schroedinger equation and the onedimensional projection operators on the Hilbert space generated by those solutions. Since such solutions to the Schroedinger equation are the pure states of quantum mechanics, an operator representation of states may be given.

In quantum mechanics, there are also mixed states. Unlike the mixed states previously discussed, they cannot be given a straightforward ignorance interpretation owing to the fact that these states do not have unique decompositions into pure states. Since the mixed states of quantum mechanics are not solutions to the Schroedinger equation, the operator representation of states is of considerable importance as it permits a representation of both pure and mixed states.

In this regard, states are represented by density operators:

Definition 1.[30] A density operator $\mathbf{W}$ is a self-adjoint positive operator such that:

1) $\Sigma_i(\emptyset_i, \mathbf{W}\emptyset_i) = 1$ for any orthonormal basis $[\emptyset_i]$.

(That is, $Tr(\mathbf{W}) = 1$).

2) $\mathbf{W} > 0$ (i.e., $\mathbf{W}\emptyset \neq 0$) unless $\emptyset$ is the zero vector.

3) $\mathbf{W}^2 \leq \mathbf{W}$ (i.e., $(\emptyset, \mathbf{W}^2\emptyset) \leq (\emptyset, \mathbf{W}\emptyset)$).

A state is a pure state in this representation just in case $\mathbf{W}^2 = \mathbf{W}$ (in which case $\mathbf{W}$ is a one-dimensional projection operator). The states for which $\mathbf{W}^2 \neq \mathbf{W}$ are the mixed states of quantum mechanics. A quantum mechanical mixed state Q_i might also be written as:

1) $Q_i = \Sigma_i \ |c_i|^2 \ \mathbf{P}_{(\emptyset i)}$,

where $\mathbf{P}_{(\emptyset i)}$ is the one dimensional projection operator corresponding to $\emptyset i$ and

2) $\Sigma_i \ |c_i|^2 = 1$

In terms of Born's probability interpretation of $|\emptyset|^2$, the expectation value for an observable A in state $\emptyset$ is understood in terms of the Hilbert space inner product:[31]

1) $E(A)_\emptyset = (\emptyset, \mathbf{A}\emptyset)$.

If $\emptyset$ is expanded:

2) $\emptyset = \Sigma_i \ c_i \ \emptyset_i$,

where $c_i = (\emptyset, \psi_i)$, and the ψ_i are the eigenvectors of $\mathbf{A}$, then:

3) $E(A)_\emptyset = \Sigma_i \ |c_i|^2 \ a_i$

where a_i is the eigenvalue corresponding to the eigenvector Ψ_i. Thus the expectation value for an observable A in quantum state $\emptyset$ is calculated as a probability weighted sum of the eigenvalues of A corresponding to the eigenvectors of **A** in the expansion of $\emptyset$. The weights $|c_i|^2$ are such that:

4) $\Sigma_i \ |c_i|^2 = 1$.

Clearly, if $\emptyset$ is an eigenvector of **A** then in state $\emptyset$ the observable A takes the corresponding eigenvalue with probability one.

As noted by Jauch,[32] in the case of a mixed state W:

1) $E(A)_{\mathbf{W}} = \mathrm{Tr}\ (\mathbf{WA})$,

and there always exists a complete orthonormal basis $[\emptyset_i]$ and positive numbers v_i such that:

2) $E(A)_{\mathbf{W}} = \Sigma_i v_i \ (\emptyset_i, \mathbf{A}\emptyset_i)$

where

3) $\Sigma_i v_i = 1$.

Two operators commute if:

1) $(\mathbf{AB} - \mathbf{BA})\emptyset = 0$ for all $\emptyset \in H$. Otherwise there is some self-adjoint operator **C** such that:

2) $(\mathbf{AB} - \mathbf{BA})\emptyset = i\ \mathbf{C}\ \emptyset$,

for all $\emptyset \in H$.

Operators do not in general commute, but if they do, then it is possible to isolate a complete orthonormal basis in H whose vectors are eigenvectors of both operators.

The question of joint probability distributions in quantum mechanics is a tricky one. Cohen[33] claims that if quantum mechanics can be formulated as a classical probability theory then there should exist a probability distribution $F(q,p)$ which satisfies:

a) $F(q,p) \geq 0$ for all q,p.

b) $F(q,p)dp = |\Psi(q)|^2$ where $\Psi(q)$ is the position state function.

c) $F(q,p)dq = |\phi(p)|^2$, where $\phi(p)$ is the momentum state function, which is

$$\left(\frac{1}{2\pi\hbar}\right)^{1/2} \int (q) \exp\left(- \frac{i}{\hbar} qp\right) dq$$

d) The expectations of observables calculated in the classical manner should be equal to those obtained using the operator formalism of quantum mechanics.

Cohen goes on to show that the most general function which satisfies (b) and (c) cannot satisfy (d). This is an important quantum mechanical result. Cohen takes as basic variables q and p, and shows that there can be no proper joint distribution for them. If position and momentum cannot have a joint distribution which is quantum mechanically adequate then one can hardly hope to recover quantum probabilities by using the random variable/phase space apparatus. Since the random variable/phase space apparatus is standardly used by hidden variables theorists, these 'no joint distribution' arguments are of considerable interest. Some of these themes will be picked up in Chapter Four. I want to make three points here. First, from a historical point of view, the 'no joint distribution' proofs have not been seen as the major threat to hidden variables theorists. Though of course, as previously noted, the claim that the Bell theorem is a 'no joint distribution' theorem of sorts will be reviewed later in this study.

Secondly there is a point, to some extent conceded by Cohen himself, that position and momentum may not be the only basic variables. Bopp[34] considers distribution functions which involve more variables than just position and momentum. To the distribution discussed by Cohen,[35] Bopp adds a variable, l, having the dimension of length. Cohen comments:

> The advantage of Bopp's distribution is that of being positive definite for all values of the variables. However, the statistical predictions obtained using F_B [Bopp's distribution] will, of course, contain l and are therefore at variance with the usual predictions of quantum theory.[36]

But as we have seen, in a hidden variables theory there may indeed be new variables. This is one reason why one should be cautious in treating the 'no joint distribution' arguments as 'no hidden variables' arguments.

Thirdly Cohen comments on the work of Bohm:

> Bohm, using the analogy of Brownian motion, has tried to explain the probabilistic nature of quantum mechanics by assuming that there exists a sub-quantum level which is governed by deterministic laws. The properties of this sub-quantum level are assumed to be of such a nature that they give rise to quantum effects much in the manner in which molecules of a classical fluid give rise to Brownian motion. Our study shows that if this program is carried through completely it must yield results which will contradict those of present day quantum mechanics in some details.[37]

While it is not appropriate here to preempt the discussion of Bohm's theory in the next Chapter, the following point can be made at this juncture. The Bohm theory is a contextual theory in which consideration is given to the different types of measurement that are possible for a given observable. It will be seen in Chapter Two that corresponding to those different measurement contexts for a given observable are distinct probability distributions. In the Bohm theory we do not retrieve the quantum probabilities from a single classical phase space distribution F(q,p). Once again, whatever the merits of Cohen's argument (a matter beyond assessment here), it is unclear that that argument is of direct relevance to all hidden variables approaches --though it is possible that some hidden variables theories do try to recover quantum probabilities from a single distribution F(q,p), it is evident that not all such approaches do.

When does quantum mechanics usually define joint distributions? According to Fine[38] quantum mechanics itself only defines joint distributions for commuting observables. If A and B commute then there will be some observable C and Borel function f such that:

1) $C = f(A,B)$.

The joint distribution will then be given as:

2) $P^{\emptyset}_{C} = (\emptyset, C\emptyset)$

where

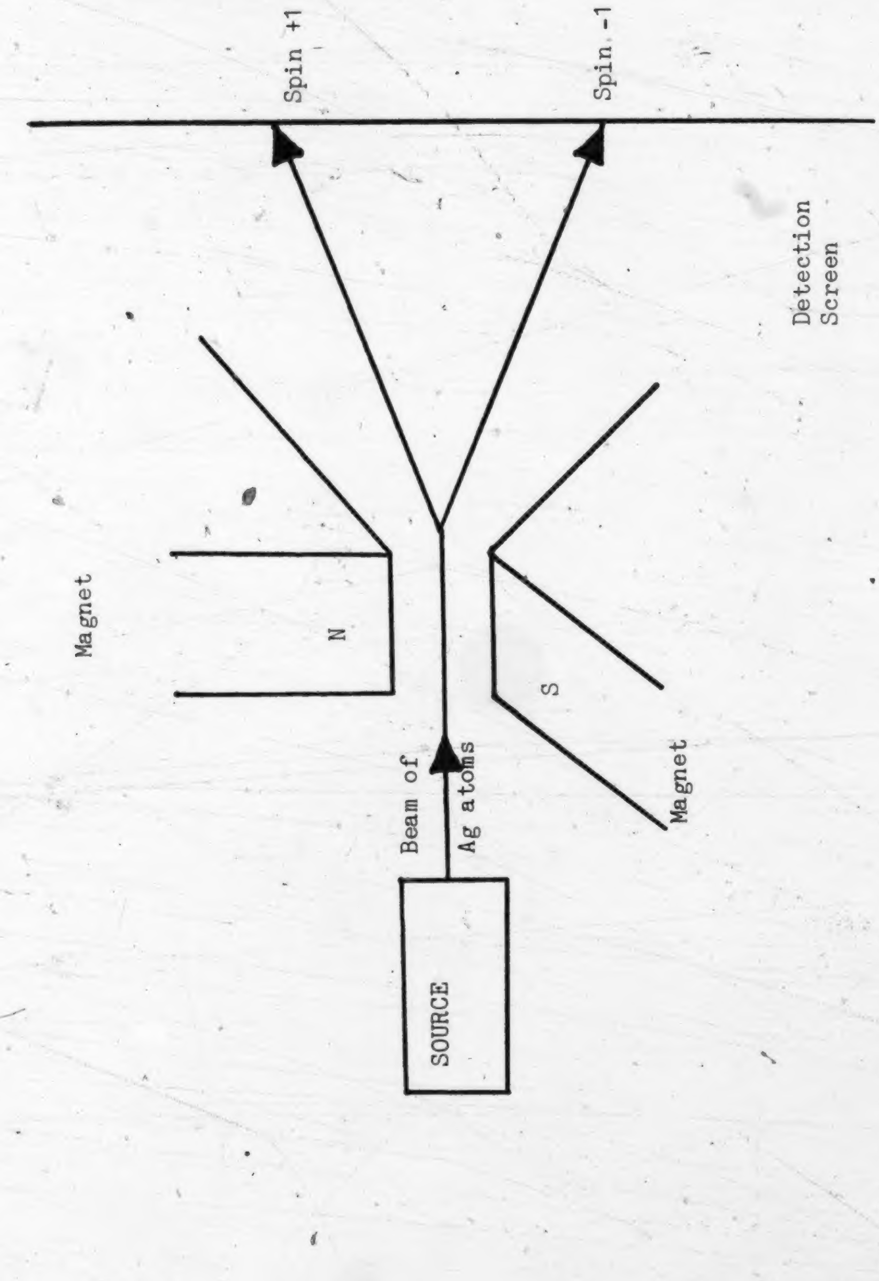
3) $P^{\emptyset}_{C} = P^{\emptyset}_{f(A,B)}$ so $P^{\emptyset}_{A,B} = P^{\emptyset}_{f^{-1}(A,B)}$.

SECTION SIX: THE CONCEPT OF SPIN IN QUANTUM MECHANICS

The quantum mechanical spin observable plays a central role in the remainder of the study. Some of its features and characteristics must now be discussed.

In much of the remainder of this study the crucial observables will be spin components[39] which may be measured using a Stern-Gerlach apparatus--see Figure 1-1. The original Stern-Gerlach

FIGURE 1-1 The Stern-Gerlach Experiment.


Spin +1
Spin -1
Detection Screen
Magnet
N
S
Magnet
Beam of Ag atoms
SOURCE

experiment was aimed at measuring (say) the vertical component of magnetic moment on silver atoms. In the experiment[40] a beam of silver atoms was passed through an inhomogeneous magnetic field. When the experiment was run, two traces appeared on the detection screen: the beam had split as a result of passing through the magnetic field. The implications of this experiment were not fully appreciated at first. It was sometime later that Goudsmit and Uhlenbeck postulated an intrinsic magnetic moment and an intrinsic spin.

This magnetic moment, along any direction $\vec{b}$, has two projections:

1) $M_{\vec{b}} = \pm \frac{e\hbar}{2\mu}$,

where μ is magnetic permeability. Though a Stern-Gerlach experiment is correctly interpreted as measuring components of intrinsic magnetic moment, it is also standardly considered as an experiment to measure components of spin by associating the two possible projections of M along any direction $\vec{b}$ with the two distinct eigenvalues for spin along that direction. Thus:

2) Spin + 1 is associated with $M_{\vec{b}} = \frac{e\hbar}{2\mu}$

Spin - 1 is associated with $M_{\vec{b}} = -\frac{e\hbar}{2\mu}$

Following Merzbacher's discussion of spin[41] consider a simple spin-system whose spin-Hilbert space $V_2(C)$, (a Hilbert space whose unit rays are spin component eigenvectors), is of two dimensions and defined over the field of complex numbers. Consider some arbitrary direction $\vec{b}$. The observable $S(\vec{b})$ will be called 'spin along $\vec{b}$'--it will correspond to some appropriate self-adjoint operator defined on $V_2(C)$. The observable $S(\overleftarrow{b})$ has eigenstates ϕ_1 and ϕ_2 corresponding to the eigenvalues $S(\overleftarrow{b}) = +1$ and $S(\overleftarrow{b}) = -1$ respectively. ϕ_1 and ϕ_2 are orthogonal and thus constitute a basis for $V_2(C)$. In addition to ϕ_1 and ϕ_2, there are states of the system which are linear superpositions of these eigenstates of $S(\overleftarrow{b})$. For example:

1) $\Psi = c_1\phi_1 + c_2\phi_2$,

where c_1 and c_2 are scalars such that:

2) $|c_1|^2 + |c_2|^2 = 1$.

For a system in state Ψ, $|c_1|^2$ gives the probability of finding $S(\overleftarrow{b}) = +1$ upon measurement. Similarly $|c_2|^2$ gives the probability of finding $S(b) = -1$.

In the Stern-Gerlach experiment referred to earlier, where $S(\overleftarrow{b})$ is being measured on individual atoms in a beam of silver atoms, we assume that each atom in the beam is initially prepared in the spin state $\Psi (= c_1\phi_1 + c_2\phi_2)$. As noted, when a beam passes through the inhomogeneous magnetic field, the beam splits into two sub-beams. One sub-beam comprises those silver atoms in the state ϕ_1, the other silver atoms in the state ϕ_2. By the projection postulate, the state of any atom immediately after measurement of $S(\overleftarrow{b})$ is to be determined on the basis of the eigenvalue revealed by measurement. Measurement of $S(\overleftarrow{b})$ on the silver atoms apparently changes the state of each atom in a discontinuous and indeterministic fashion: all that is known for any individual atom in the beam is that there is a probability of .5 that it will undergo a state transition from Ψ to ϕ_1, and a probability of .5 that it will undergo a transition from Ψ to ϕ_2.

The operators corresponding to distinct components of spin on our system do not commute. This leads Merzbacher to comment:

> For example, $S(\overleftarrow{z})$ and $S(\overleftarrow{y})$ are incompatible, for they do not commute; a state cannot simultaneously have a definite value of $S(\overleftarrow{z})$ and $S(\overleftarrow{y})$. If we wish to measure $S(\overleftarrow{z})$ and $S(\overleftarrow{y})$ for a state ϕ, two separate ensembles must be used. The two components of spin cannot be measured simultaneously on the same system.[42]

It is true that $S(\overleftarrow{z})$ and $S(\overleftarrow{y})$ do not have eigenstates in common. Whether this precludes their taking simultaneous exact values is a question of fundamental importance to the hidden variables issue. Merzbacher, of course, takes the orthodox line concerning these matters.

Spin systems may consist of more than one particle.[43] Consider a two-particle spin system. Label the component systems 1 and 2 respectively. Associated with each of the particles will be Hilbert spaces H_1 and H_2, (each being like $V_2(C)$). In order to discuss the two-particle system 1 + 2 we use the Hilbert space $H = H_1 \otimes H_2$, which is the four-dimensional tensor product space of the component Hilbert spaces.

An important observable on 1 + 2 is the total spin:[44] $S = S_1 + S_2$. The total spin S_1 of 1 commutes with the total spin S_2 of 2. An important state of this product space observable S is the singlet spin state corresponding to S=0. This particular solution to the Schroedinger equation will play a role in Chapter Three in the discussion of Bell's theorem. The singlet spin state ψ_0 may be represented as follows:

$$2)\ \psi_0 = \frac{1}{\sqrt{2}}[\phi_1(+1)\phi_2(-1)-\phi_1(-1)\phi_2(+1)]$$

A pair-system 1 + 2 prepared in this state and otherwise undisturbed will remain in this state until a measurement is performed on either 1 or 2. If spin is measured along some direction $\vec{b}$ on 1, and the value +1 is found, then by the projection postulate, immediately after measurement, the state of the pair-system is:

3) $\phi_1(+1)\phi_2(-1)$

corresponding to $S_1(\vec{b}) = +1$ and $S_2(\vec{b}) = -1$. The reduction of ψ_0 is inherently probabilistic and we might just as well have found:

4) $\phi_1(-1)\phi_2(+1)$

Either way, it can be seen that the spin component measurement values along any direction are anti-correlated for 1 + 2. Pair of particles in the singlet spin state are the systems of concern in Bell's theorem.

FOOTNOTES

[1] Schroedinger on quantum mechanics. Quoted in the opening of Gribbin (1984).

[2] E.g., Bohm and Hiley (1981).

[3] E.g., Ballentine's statistical interpretation (1970).

[4] E.g., Boyer's work on stochastic electrodynamics (1975).

[5] See Jammer (1974).

[6] There is, for instance, much that Bohr and Heisenberg agree on, but there are significant differences between their interpretations of quantum mechanics. See Jammer (1974), Chapter Three.

[7] Jammer (1974), p. 248.

[8] Bohr (1958), p. 311. See also Hooker (1972), Part II.

[9] According to the Copenhagen Interpretation, this is the experimental significance of the non-commutativity of position and momentum.

[10] Einstein (1957), p. 31.

[11] Fine (1974), p. 740.

[12] The two-slit experiment is discussed in Chapter Two of this study.

[13] Heisenberg (1930), p. 23.

[14] Ibid., p. 36.

[15] Merzbacher (1961), p. 10.

[16] Ibid., p. 11.

[17] Bohm (1951), pp. 117-118.

[18] See Fine (1974) for a discussion of the eigenstate-eigenvalue link. See also Cartwright (1983), p. 168.

[19] Lorentz (1937), pp. 219-223.

[20] See Powers (1982), Chapter three for a discussion of aether theories and the Michelson-Morley experiment.

[21] The reduction of phenomenological thermodynamics to classical mechanics is a classic precedent more from the point of view of the analysis of the probabilistic element of a physical theory in terms of the random variable/phase space apparatus, than from the point of view of the reduction of one physical theory to another. See Kochen and Specker (1967), p. 57. It turns out that the reduction question is nevertheless an interesting one that has some bearing on certain hidden variables theories. For instance, the theory of stochastic electrodynamics, which is discussed in Chapter Two of this study, tries to provide an electrodynamical understanding of quantum mechanics.

[22] Tolman (1967), p. 524.

[23] In the Fitzgerald-Lorentz contraction theory, the postulated contractions of the apparatus were not merely geometric effects as in the later Einstein theory of special relativity. Rather, the apparatus contractions were viewed as dynamical effects of motion through the aether.

[24] E.g., Bell (1964). Bell uses hidden variables in this sense.

[25] Bell (1964).

[26] See Reif (1965), p. 51.

[27] This definition is a version of Fine's definition (1982), p. 1306.

[28] From a notational point of view, I will distinguish between observables and operators in the following way: the symbol A will represent an observable whereas the symbol **A** will represent an operator. One point to note is that the symbol H stands for Hilbert space whereas **H** stands for the Hamiltonian operator.

[29] A subset M of a Hilbert space H is a linear manifold if it is closed under vector addition and multiplication by scalars.

[30] Jauch (1968), p. 132.

[31] This treatment of expectation values follows Sharp (1978), pp. 40-41.

[32] Jauch (1968), p. 132.

[33] Cohen (1966), p. 317.

[34] Bopp (1957), pp. 189-196.

[35] Cohen (1966), p. 321.

[36] Ibid., p. 322.

[37] Ibid., p. 321.

[38] Fine (1982), p. 1309.

[39] The spin components are components of the total spin on the system concerned. All spin components commute with the total spin observable on the system of which they are components. These spin components typically do not commute with each other.

[40] See Merzbacher (1961), pp. 248-253.

[41] Ibid., p. 251.

[42] Ibid., p. 279. (Notation altered).

[43] As was seen in Section One of this Chapter, talk of particles seems to require talk of simultaneous exact values for all observables. It remains true that physicists sometimes deny that all observables on a system take simultaneous exact values and nevertheless refer to that system as a particle. One wonders whether this is coherent.

[44] See note (39).

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CHAPTER TWO

THE HIDDEN VARIABLES QUESTION IN QUANTUM MECHANICS

> The discussion so far has assumed that the electron has no position value because it is in a superposition of position states. We have thus been adopting the principle: S has a value for a given observable (an eigenvalue) if and only if S is in the corresponding state (the eigenstate). To deny the inference in the direction from state to value would require serious revision of our understanding of quantum theory . . . But there is no harm in breaking the inference in the other direction. One must be careful not to run foul of the various no-hidden-variable proofs like those of Kochen and Specker and of J.S. Bell. But there are a variety of satisfactory ways of assigning values to systems in superpositions.
>
> N.D. Cartwright[1]

SECTION ONE: INTRODUCTION

It was seen in the last Chapter that no quantum state determines simultaneous exact values for all quantum mechanical observables. Quantum states are thus not classically determinate states. The minimal aspect of the hidden variables issue which has typically engaged theorists in this area, (as was seen in Chapter One), is whether quantum mechanics can be interpreted in terms of states which are classically determinate. Such states would assign simultaneous exact values to all quantum mechanical observables. Suppose quantum mechanics can be interpreted in terms of clasically determinate states. It would not follow from this that quantum mechanics was a classical theory. That notion requires, in addition, that quantum mechanics satisfy further criteria of descriptive adequacy such as completeness of description of physical systems. Nevertheless, a necessary condition for quantum mechanics to be a classical theory is that it be interpretable in terms of classically determinate states. Correspondingly, the aim of the 'no hidden variables' proofs has been to establish that it is inconsistent with quantum mechanics to assume that all quantum mechanical observables on a system take simultaneous exact values at any time.

In this Chapter a variety of hidden variables strategies will be presented. In particular, the two major classes of hidden variables strategy, non-contextual and contextual, will be differentiated. In addition, two algebraic 'no hidden variables' proofs will be discussed with a view to showing that some hidden variables strategies avoid the consequences of these proofs. Finally, two examples of a hidden variables theory will be presented to illustrate some of the concepts discussed earlier in the Chapter.[2]

SECTION TWO: NON-CONTEXTUAL HIDDEN VARIABLES STRATEGIES

Non-contextual hidden variables strategies may be contrasted with the contextual strategies discussed in Section Four below. The aim of a non-contextual hidden variables theory is to interpret quantum mechanics in terms of classically determinate states through a consideration of quantum mechanical observables alone. The classically determinate states of a non-contextual hidden variables theory will be lists of these values. Each such state may be viewed either as a list of measurement-independent observable values or as a list of values that would be found were measurements to be performed. Realists have typically been interested in the former interpretation of a classically determinate state whereas anti-realists (as well as those who wish to remain neutral in the dispute between realists and anti-realists) have opted for the latter interpretation of a classically determinate state.

As this Chapter proceeds, it will quickly emerge that the issue as to whether quantum mechanics can be given a hidden variables interpretation depends on what constraints are placed on the production of such an interpretation. The 'no hidden variables' proofs examined in this Chapter suggest that there are constraints on the production of a hidden variables interpretation of quantum mechanics which cannot possibly be satisfied. The standard rejoinders to these proofs consist typically of arguments to the effect that these constraints are not reasonable or will not accomplish all that they are intended to accomplish.

The first hidden variables strategy to be discussed--HV-1--is extremely simple. HV-1 is the hidden variables strategy that is typically invoked in discussions of the hidden variables question.[3]

HV-1

This simple hidden variables strategy can be represented by the following axioms:

1) Each quantum system normally described by the quantum state vector $\emptyset$ will be described instead by classically determinate states λ in a phase space Λ.

Thus if $A_1, \ldots, A_n, \ldots$ are quantum mechanical observables then each $\lambda \in \Lambda$ is essentially a list $\langle a_1, \ldots, a_n, \ldots \rangle$ of values for those observables. As noted before, these values may be measurement-independent observable values or they may be values that would be found were certain measurements to be performed.

2) Each quantum state $\emptyset$ is associated with a phase space measure $\Delta_{\emptyset}$ on the measurable subsets of Λ such that if Γ is a measurable

subset of Λ, then $\Delta_{\emptyset}(\Gamma)$ gives the probability that the classically determinate state λ of the system lies in Γ.

3) Each quantum mechanical observable A is associated with a unique real-valued measurable function $f_A: \Lambda \rightarrow R$.

HV-1 respects the following constraint:

Constraint 1. If M is a Borel set of real numbers and $P_A^{\emptyset}$ is the quantum mechanically well-defined probability measure on the real line R associated with quantum observable A and quantum state $\emptyset$ such that $P_A^{\emptyset}(M)$ is the probability that $Val(A)_{\emptyset} \in M$,[4] then:

1) $P_A^{\emptyset}(M) = \Delta_{\emptyset}(f_A{}^{-1}(M))$

2) $E(A)_{\emptyset} = \int_{\Lambda} f_A(\lambda) d\Delta_{\emptyset}$.

Kochen and Specker[5] show that HV-1 can satisfy Constraint 1 by treating the quantum mechanical observables as independent random variables. Thus, if the only constraint on the production of a hidden variables interpretation of quantum mechanics is Constraint 1 then quantum mechanics certainly admits of a hidden variables interpretation.

In von Neumann's argument[6] against the possibility of hidden variables interpretations of quantum mechanics, it is suggested that there are other constraints which must be taken into account. As stated by Jammer[7] these constraints are:

Constraint 2. If an observable A is represented by some appropriate self-adjoint operator **A** then a function g of this observable will be represented by the operator g(**A**).

In terms of the associated random variables, if observable A is represented by f_A, then the observable g(A) will be represented by $g(f_A)$.

Constraint 3. If observables A, B, . . . are represented by operators **A**, **B**, . . . then the sum of these observables will be presented by the operator **A**+**B**+ . . ., regardless of whether these operators commute or not.

In terms of the associated random variables, the sum of the observables will be represented by $f_A + f_B + \ldots$

Constraint 4. If the observable A is non-negative, then its expectation value $E(A)_\phi$ will be non-negative.

In terms of the associated random variable f_A, $E(f_A)_\lambda$ will be non-negative.

Constraint 5. If A, B, . . . are arbitrary observables and a, b, . . . are real numbers, then:

$$E(aA + bB + \ldots)_\phi = aE(A)_\phi + bE(B)_\phi + \ldots$$

In terms of the associated random variables, this constraint amounts to the requirement that even for the classically determinate (dispersion-free) states λ there must be additivity of expectation values.

According to von Neumann, HV-1 cannot satisfy constraints (1)-(5). This is not the place to discuss either the details of the von Neumann argument or the enormous literature on it. (The interested reader should consult Jammer[8]). The following two comments will suffice.

First, Bell[9] has pointed out that Constraint 5 is a special property of quantum states and there is no reason to suppose that it must be satisfied for the classically determinate (dispersion-free) states ©. Briefly, Bell reasons that if A and B are noncommuting observables then the apparatus required to measure A + B will in general be different from that required to measure either A or B.[10] Bell concludes that there is no a priori reason to accept Constraint 5 as regards the statistical relations between the corresponding measurement results.

Second, as noted by van Fraassen,[11] instead of Constraint 5, and given that each operator **A** is associated with an infinite matrix $(\mathbf{A})_{ij}$, von Neumann could have chosen:

Constraint 5(a)

$$(a\mathbf{A} + b\mathbf{B} + \ldots)_{ij} = a(\mathbf{A})_{ij} + b(\mathbf{B})_{ij} + \ldots$$

Given suitable definitions, Constraint 5 can be derived as a theorem from Constraint 5(a). Thus derived it will concern expectation values and quantum states only, (from which von Neumann's 'no hidden variables' proof does not follow). As pointed out by van Fraassen,[12] no hidden variables strategy can accept Constraint 5, however, Constraint 5(a) would be quite acceptable.

Kochen and Specker,[13] in their 'no hidden variables' proof, try to avoid making assumptions about non-commuting observables. They thus hope to avoid some of the objections to the von Neumann proof. They suggest that hidden variables strategies should satisfy the following constraint: for any set of compatible (i.e., pairwise commuting) observables, the values of these observables must stand in the same functional relations as do the corresponding Hilbert space operators. In terms of the associated random variables, if A and g(A) are observables corresponding to operators **A** and g(**A**), then it is required that the random variables f_A and $g(f_A)$ corresponding to these observables in HV-1 be such that:

(K and S) $f_{g(A)}(\lambda) = g(f_A(\lambda))$

for any λ and Borel function g.

(K and S) differs from Constraint 5 in that it concerns commuting observables only, but also in that it concerns values of observables and not merely their expectation values. Kochen and Specker offer a proof that if the Hilbert space associated with a quantum system is of three dimensions or more, then there can be no hidden variables strategy for the observables on that system.

Briefly put, the Kochen and Specker argument proceeds by turning (K and S) into the following algebraic embedding requirement: a necessary condition for HV-1 to satisfy (K and S) is that the partial algebra Y of quantum projection operators[14] be embeddable into a Boolean algebra B.

Necessary and sufficient for this embedding to be possible is that for every pair of distinct elements v, w $\in$ Y, there exist a homomorphism $h: Y \rightarrow Z_2$, (Z_2 a Boolean algebra over a field of two elements), such that $h(w) \neq h(v)$. Kochen and Specker then show, by a consideration of 117 unit vectors in $V_3(R)$[15] that no such homomorphism is possible. In essence, they argue that though each maximal Boolean sub-algebra of the partial Boolean algebra Y can be taken homomorphically onto Z_2, it is nevertheless the case that the union of such homomorphisms will not be a homomorphism. Kochen and Specker illustrate the physical relevance of their argument by a consideration of an orthohelium atom in its lowest triplet state. They consider 117 components of spin on this system and demonstrate that values for these spin components cannot be distributed in a manner consistent with the statistical predictions of quantum mechanics.

The important question here is whether (K and S) is a reasonable constraint on a hidden variables strategy.[16] To this end, Kochen and Specker's comments on (K and S) turn out to be important. First, they define the observable g(A), for every observable A and Borel function g, by their formula (3):

(K and S-3) $P^{\emptyset}_{g(A)}(M) = P^{\emptyset}_{A}(g^{-1}(M))$,

for each state Ø.

They comment:

> If we assume that every observable is determined by the function P, i.e., $P^{\emptyset}_{A} = P^{\emptyset}_{B}$ for every Ø implies A=B, then the formula (3) defines the observable g(A).[17]

Revealingly, they add:

> Thus the measurement of a function g(A) of an observable A is independent of the theory considered--one merely writes g(a) for the value of g(A) if a is the measured value of A. The set of observables of a theory thereby acquires an algebraic structure, and the introduction of hidden variables into a theory should preserve this structure.[18]

Clearly, inasmuch as there is an argument here for (K and S), it begs the question, for the claim quoted above assumes (K and S) holds--for otherwise one could not measure g(A) by measuring A and applying g to the resultant value.[19]

In the next two sections, ways in which (K and S) can fail will be presented and discussed.

SECTION THREE: SHARP AND THE STATISTICAL INTERPRETATION OF QUANTUM MECHANICS

The central concern of Sharp's study[20] is the tenability of the Statistical Interpretation of quantum mechanics.[21] The central difference between orthodox interpretations of quantum mechanics and the Statistical Interpretation derives from the respective conceptions of the pure states of quantum mechanics. In orthodox quantum mechanics, the pure states are taken to provide complete descriptions of individual systems. In the Statistical Interpretation the pure states are taken to describe certain statistical properties of ensembles of systems. In what follows, criteria for associating states and ensembles will turn out to be important. Such criteria will be referred to as ensemble membership criteria. The Statistical Interpretation of quantum mechanics is of interest to the hidden variables strategist since--and this is one of the main points in Sharp's work--implicit within it is the claim that all observables on any quantum system take simultaneous exact values. These values will not, of course, be determined by any quantum state $\emptyset$. Nevertheless, implicit within the Statistical Interpretation is reference to classically determinate states.

If quantum states $\emptyset$ are to be associated with ensembles $E_{(\emptyset)}$ of quantum systems then, minimally, some ensemble-membership criterion will be needed. An ensemble membership criterion is a criterion for assigning systems to ensembles representing a particular state. As will be seen below, corresponding to different ensemble membership criteria are different versions of the Statistical Interpretation.

In discussing the Statistical Interpretation of quantum mechanics, and in order to bring out its implicit 'hidden variables' character, it will be useful to restate axiom-1 of HV-1 as follows:

1a) Each ensemble $E_{(\emptyset)}$ of quantum systems normally represented by the quantum state vector $\emptyset$ will be represented instead by a phase space Λ. To each system x in $E_{(\emptyset)}$ there will correspond an element $\lambda \in \Lambda$, where λ is a list of values for all observables ascribed to x by quantum mechanics.

(a) HV-1 and the Preparation Criterion

Before it is possible to have an ensemble interpretation of quantum states, some criterion is necessary by which systems can be assigned to statistical ensembles. In one recent proposal due to Ballentine,[22] among others, use is made of a certain notion of 'preparation'.

Ballentine's ensembles, for example, are to be ensembles of 'similarly prepared' systems. Ballentine notes with reference to single electron systems:

> . . . a momentum eigenstate . . . represents the ensemble whose members are single electrons each having the same momentum, but being dispersed uniformly over all positions.[23]

This suggests that a statement of an ensemble-membership criterion can proceed in terms of possession of appropriate eigenvalues for certain observables. Sharp suggests that the following criterion is employed by Ballentine:[24]

Preparation Criterion. A system belongs to $E_{(\emptyset_i)}$ if it has been prepared to have eigenvalue a_i for observable A, where $\emptyset_i$ is the eigenstate of A corresponding to the eigervalue a_i.

For example, systems can presumably be prepared to belong to an ensemble $E_{(\emptyset)}$, where $\emptyset$ corresponds to spin +1 along $\vec{z}$, by subjecting them to a Stern-Gerlach experiment to measure spin along $\vec{z}$. A system will belong to $E_{(\emptyset)}$ if it is a member of the appropriate beam--the 'spin +1 along $\vec{z}$'-beam--as a result of measuring the $\vec{z}$-component of spin.

One should not lose sight of the fact, as Sharp notes, that the notion of 'preparation' is at best somewhat vague and at worst irredeemably anthropocentric. That, however, is the least of our worries in the present context.

The statistical hidden variables strategy we are considering here consists of axioms (1a), (2), and (3) and use of the Preparation Criterion for ensemble-membership. If this hidden variables strategy is to avoid the disastrous consequences of the Kochen and Specker argument, then use of the Preparation Criterion must not imply that (K and S) is satisfied.

Does use of the Preparation Criterion imply that (K and S) is satisfied? This matter can be elucidated by a consideration of certain algebraic constraints imposed on the members of any ensemble. In particular, if $\emptyset_i$ is the eigenstate of A corresponding to the eigenvalue a_i, then any member of $E_{(\emptyset_i)}$ must have not only the value a_i for A but also the value of $g(a_i)$ for any observable $g(A)$, since $\emptyset_i$ is also an eigenstate of any observable $g(A)$ with eigenvalue $g(a_i)$. That is, the functional relations which exist in the Boolean subalgebra generated by A must be satisfied by any member of $E_{(\emptyset_i)}$. If the present hidden variables strategy with the Preparation Criterion is to be defensible, then there must be a legitimate way to avoid the generalization to other contexts of the algebraic constraints which are placed on a system when is satisfies some criterion for ensemble membership.

At this point, Arthur Fine's advice[25] not to generalize relations forced out by eigenstates is relevant. Consider an observable A and a system x_i. If x_i belongs to $E(\phi_i)$, then it is known that it has eigenvalue a_i for A, where ϕ_i is the eigenstate of A corresponding to the eigenvalue a_i and, as noted above, the value $g(a_i)$ for $g(A)$, for any Borel function g. Consider now a system x_j belonging to the ensemble $E(\psi_i)$, where:

1) $\psi_i = \Sigma_i c_i \phi_i$,

for $[\phi_i]$ a complete orthonormal set of A-eigenstates. On the present view the system x_j will belong to $E(\psi_i)$ in virtue of being prepared to possess an eigenvalue b_i of observable B (which does not commute with A).

In order to avoid the force of the Kochen and Specker argument, one will want the observable A on x_j to take some one of its eigenvalues, and similarly one will want any observable g(A) on x_j to take some one of its eigenvalues. Now since ψ_i is not an eigenstate of A or of any observable g(A), quantum mechanics says nothing conclusive concerning algebraic relations between the values of A and any observable g(A). If we did generalize the relations forced out by eigenstates then we would conclude, perhaps, that Val(g(A)) = g(Val(A))--but in the present case quantum mechanics does not force this upon us.[26]

Thus it may be the case that an element $\lambda \in \Lambda$ assigns a value a_i to A and a value a_j to an observable g(A) where $g(a_i) \neq a_j$. Thus the present hidden variables strategy may avoid the Kochen and Specker argument.

(b) HV-1 and the Statistical Eigenvalue Criterion

Sharp[27] has explored the possibility of constructing a hidden variables strategy which uses, instead of the preparation criterion, the following ensemble-membership criterion:

The Statistical Eigenvalue Criterion: A system belongs to $E_{(\emptyset_i)}$ if it is chosen at random and has value a_i for A, where $\emptyset_i$ is the eigenstate of A corresponding to a_i.

Use of this membership criterion implies that the ensemble $E_{(\emptyset_i)}$ is just a random sample of systems which have the value a_i for the observable A. Sharp notes that hidden variables strategies which employ the statistical eigenvalue criterion cannot succeed since the statistical eigenvalue criterion implies (K and S). As Sharp puts it:

> Suppose, contrary to the Kochen and Specker constraint, that there are systems with the value a for A, but a value other than g(a) for g(A). Letting '$\emptyset$a' denote the eigenfunction of A corresponding to eigenvalue a, we know that the ensemble $E_{(\emptyset a)}$ is such that all its members have value g(a) for g(A). But if $E_{(\emptyset a)}$ is, as the statistical eigenvalue criterion would have it, just a rahdom sample of systems with value a for A, then some of its members, by our assumption, would have a value other than g(a) for g(A). Thus we must reject the assumption and accept the Kochen and Specker constraint.[28]

(c) HV-1 and the Fortuitous Statistical Eigenvalue Criterion

As an alternative to the disastrous ensemble membership criterion just discussed, Sharp has offered the following:[29]

The Fortuitous Statistical Eigenvalue Criterion: A system belongs to the ensemble $E_{(\emptyset_i)}$ if it is chosen at random and has the value a for A and the value g(a) for any observable g(A), where $\emptyset$ is the eigenstate of A corresponding to the eigenvalue a.

This ensemble-membership criterion implies that the ensemble $E(\emptyset_i)$ is a random sample of systems which (fortuitously) just happen to satisfy the algebraic constraints on ensemble membership. According to this criterion, there are systems--those not chosen, even at random--which have a value a for A but a value other than g(a) for g(A). HV-1 based on the fortuitous statistical eigenvalue criterion can avoid the disastrous consequences of the Kochen and Specker argument. As Sharp notes, the price is the clearly ad hoc nature of the statistical samples.

It would be desirable to have some plausible theoretical account of why systems chosen at random just happen to satisfy (K and S), when there are others--those not chosen, even at random--which violate (K and S). To the best of the present author's knowledge we do not possess such an account. Still, if general conclusions are being contemplated, then the albeit fortuitous hidden variables strategy should not be overlooked.

So far we have explored the non-contextual hidden variables strategy HV-1. The 'no hidden variables' proofs of von Neumann and Kochen and Specker have been examined. It has been seen that these arguments hinge crucially on the imposition of constraints which the hidden variables theorist cannot hope to satisfy. Whether those constraints are reasonable is an important matter. It is widely agreed that the von Neumann constraint--Constraint 5 above--is unreasonable. In connection with (K and S) however, there is a variety of perspectives. In the discussion of statistical versions of HV-1, it was seen that certain ensemble-membership criteria apparently did not require or imply (K and S) whereas one--the statistical eigenvalue criterion--did.

In the next section a variety of contextual hidden variables strategies will be explored. In connection with these strategies, the constraint (K and S) will be found to be unreasonable.

SECTION FOUR: CONTEXTUAL HIDDEN VARIABLES STRATEGIES

An often-voiced view (concerning quantum mechanics) is that it gives a special role to the concept of measurement. This is certainly an integral part of Bohr's philosophy of quantum mechanics. Commenting on the role of the measuring apparatus, Bohr noted:

> The crucial point . . . implies the impossibility of any sharp separation between the behaviour of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear.[30]

The hidden variables strategies to be discussed below display sensitivity to the multiplicity of ways of measuring a given quantum observable. It will emerge that in these strategies we cannot discuss such things as values for observables, distributions of hidden classically determinate states and the correspondence between observables and random variables--or relate these matters to quantum mechanical predictions--without reference to "measurement contexts". Reference to measurement contexts, as additional factors in a hidden variables analysis, is what differentiates contextual from non-contextual hidden variables strategies. But first, just what is a measurement context?

In orthodox quantum mechanical lore one can measure simultaneously any maximal set of commuting observables. This suggests a convenient theoretical analysis of a quantum measurement context as follows: each measurement context can be associated with a complete orthonormal set of eigenstates of the measured observable. The set of measurement contexts is thus in one-one correspondence with the set of orthonormal bases in the Hilbert space associated with the system under analysis.

Using such a concept of 'context' one can already see, in an intuitive fashion, how the Kochen and Specker argument can be subverted. Consider the vector space $V_3(R)$. The vectors ϕ_1, ϕ_2, ϕ_3 and ϕ_1, ψ_2, ψ_3 form two distinct orthonormal bases for $V_3(R)$ (see Figure 2-1). Associated with those vectors will be one-dimensional projection operators:

$\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$ and $\mathbf{P}_1$, $\mathbf{Q}_2$, $\mathbf{Q}_3$.[31]

Each orthonormal basis for $V_3(R)$ will correspond to a maximal Boolean sub-algebra of the partial Boolean algebra of projection operators. The projection operators in each distinct maximal Boolean sub-algebra will commute in pairs and may each be written as Borel functions of a maximal operator thus:

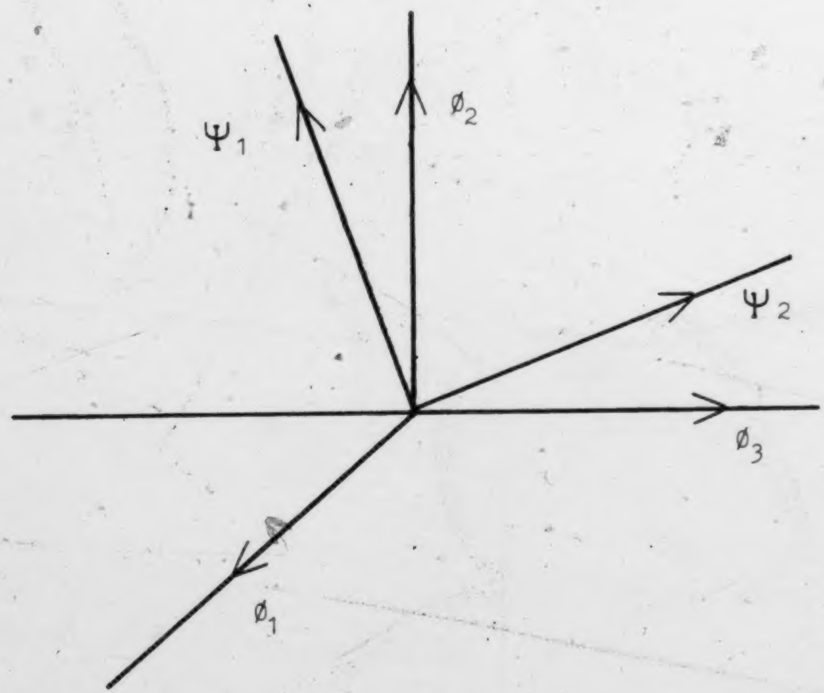
1) $\mathbf{P}_1 = g_1(\mathbf{A})$, $\mathbf{P}_2 = g_2(\mathbf{A})$, $\mathbf{P}_3 = g_3(\mathbf{A})$;

$\mathbf{P}_1 = h_1(\mathbf{B})$, $\mathbf{Q}_2 = h_2(\mathbf{B})$, $\mathbf{Q}_3 = h_3(\mathbf{B})$,

for Borel functions g_i and h_i $(i = 1,2,3)$ and maximal operators $\mathbf{A}$, $\mathbf{B}$ ($\mathbf{A}$ with eigenstates ϕ_1, ϕ_2, ϕ_3; $\mathbf{B}$ with eigenstates ϕ_1, ψ_2, ψ_3).

The non-contextual response to the constraint (K and S) sought to subvert the claim that val(g(A))) = g(val(A)). However, in contextual theories we might allow that one way to measure $\mathbf{P}_1$, $\mathbf{P}_2$

FIGURE 2-1 Two Orthonormal Bases for $V_3(R)$.


Ψ_1
$\emptyset_2$
Ψ_2
$\emptyset_3$
$\emptyset_1$

and P_3 is to measure the observable corresponding to the maximal operator **A** and apply the appropriate Borel functions to the measured value of this observable. That is, for any particular measurement context we accept the claim that val(g(A)) = g(val(A)).

The contextual critique of (K and S) questions the applicability of (K and S) across measurement contexts so that the result of measuring P_1 when it is considered as a member of one maximal set of commuting observables, will remain the same when it is considered as a member of another maximal set of commuting observables. Bell has noted that this need not be the case in a theory sensitive to measurement contexts:

> It was tacitly assumed that measurement of an observable must yield the same value independently of what other measurements may be made simultaneously . . . the result of an observation may reasonably depend not only on the state of the system (including hidden variables) but also on the complete disposition of the apparatus.[32]

Essentially, Bell's contextual claim is this: since maximal operators **A** and **B** do not commute[33] then their measurement, according to standard quantum mechanical lore, will require different experimental set-ups (i.e., distinct configurations of the apparatus or distinct apparatuses). A context which is good for the measurement of the observable corresponding to **A** will not be good for the measurement of the observable corresponding to **B**. Hence there is no reason to suppose that the value found for the observable corresponding to P_1 in one measurement context will be the same as that found in another measurement context. A contextual hidden variables strategy in which the value of P_1 may vary from context to context shows sensitivity to the different measurements that may be done of P_1.

There are essentially four ways of setting up contextual hidden variables strategies.

(a) HV-2: The One-Many Observable Correspondence Approach

In this hidden variables strategy sensitivity to measurement contexts--to the types of measurement that may be performed for a given observable--is displayed through the relation between observables and random variables. HV-2 consists of the following three axioms:

1) Each quantum system normally described by a quantum state $\emptyset$ will be described instead by classically determinate states $\lambda \in \Lambda$.

2) Each quantum state $\emptyset$ will be associated with a unique phase space measure $\Delta_\emptyset$ on the measurable subsets of Λ.

3) For each quantum observable A and measurement context c there will be a unique random variable $f_{(Ac)}: \Lambda \longrightarrow R$.

Particularly noteworthy is axiom (3) which associates with each observable A, a family $[f_{(Ac)}]$ of random variables on Λ. Recalling Bell's response[34] to arguments like Kochen and Specker's, the observable corresponding to P_1 will, in the case discussed above, be associated with a family of random variables, one for each measurement context. Figure 2-1, given the association of contexts with orthonormal bases, illustrates two contexts c_i and c_j. Of particular interest in this case will be the two random variables $f_{(P_1c_i)}$ and $f_{(P_1c_j)}$.

Kochen and Specker, in the course of their argument against hidden variables, consider 117 projection operators P_i. They argue that in order to provide values for all of these operators in accord with quantum mechanics, some projection operator P_i must receive contradictory values a_i and a_j. In HV-2 there is no problem here, for the values a_i and a_j are values of distinct random variables:

4) $f_{(P_1c_i)}(\lambda) = a_i$, $f_{(P_1c_j)}(\lambda) = a_j$.

The probabilistic constraint on HV-2 is as follows:

Constraint C2. If M is a Borel set of reals and $P^{\emptyset}_{A}(M)$ is the probability in state $\emptyset$ that $\mathrm{Val}(A) \in M$, then relative to a context c:

a) $P^{\emptyset}_{A}(M) = \Delta_{\emptyset}(f^{-1}_{(Ac)}(M))$

b) $E(A)_{\emptyset} = \int_{\Lambda} f_{(Ac)}(\lambda)\, d\Delta_{\emptyset}$

In discussing Bell's response to the Kochen and Specker argument above, it was seen that $\mathbf{P}_1$ was represented by Borel functions of two distinct maximal operators $g_1(\mathbf{A})$ and $h_1(\mathbf{B})$.

In HV-2, $g_1(\mathbf{A})$ is associated with random variable $f_{(P_1 c_i)}$ and $h_1(\mathbf{B})$ is associated with random variable $f_{(P_1 c_j)}$.

In HV-2 the following statistical equivalence must be satisfied in order to satisfy the statistical predictions of quantum mechanics:

1) $P^{\emptyset}_{g_1(A)}(M) = P^{\emptyset}_{h_1(B)}(M)$

$= \Delta_{\emptyset}(f^{-1}_{(P_1 c_i)}(M)) = \Delta_{\emptyset}(f^{-1}_{(P_1 c_j)}(M))$

so that

2) $E(g_1(A))_{\emptyset} = E(h_1(B))_{\emptyset}$

$$= \int_{\Lambda} f_{(P_1 c_i)}(\lambda)\, d\Delta_{\emptyset} = \int_{\Lambda} f_{(P_1 d_j)}(\lambda)\, d\Delta_{\emptyset}$$

This statistical equivalence may be satisfied without it being the case that $f_{(P_1 c_i)}(\lambda) = f_{(P_1 c_j)}(\lambda)$.

(b) HV-3: The Relational State Approach

Shimony has commented:

> A contextual hidden variable theory differs from classical physics by regarding as relational many features which in classical physics . . . are thought to be intrinsic to (a system) S.[35]

In HV-3, sensitivity to the many measurement contexts which are associated with a given observable is cashed out as follows: properties traditionally viewed a properties of a micro-system alone become viewed as relations between a micro-system (e.g., an electron) and a macro-system (e.g., a photographic plate). HV-3 may be understood in terms of the following four axioms:

Let Λ be a set of micro-system states λ and let π be a set of measurement contexts c.

1) Let the phase space $\mu = \Lambda \times \pi$ be such that its elements are pairs (λ, c).

2) Let systems normally described by quantum states $\emptyset$ be described instead by classically determinate relational states $(\lambda, c) \in \mu$.

3) Corresponding to each quantum observable A there is a unique random variable $f_A : \mu \rightarrow R$.

4) With each quantum state $\emptyset$ let there be associated a unique probability measure $\Delta_{\emptyset}$ on the measurable subsets of μ.

The probability measure $\Delta_{\emptyset}$ must satisfy:

Constraint C3.

a) $P^{\emptyset}_{A}(M) = \Delta_{\emptyset}(f_A{}^{-1}(M))$

b) $E(A)_{\emptyset} = \int_{\Lambda} f_A(\lambda, c)\, d\Delta_{\emptyset}$

In HV-3, where the quantum observables are analyzed as relational properties between micro-systems and macro-systems, the disastrous consequences of the Kochen and specker argument are avoided because there is no reason to suppose that

1) $f_A(\lambda, c_i) = f_A(\lambda, c_j)$, for $(i \neq j)$.

The Kochen and Specker argument considers hidden variables strategies which are insensitive to measurement contexts.

(c) HV-4: The Total State Approach

This approach is similar to HV-3, but there is an important conceptual difference. In HV-3 properties were treated as relations. The relata were the micro-system and the macro-system. The observables were two-place functions and we spoke of composite systems each consisting of a micro-system and a macro-system.

HV-4, however, reflects the feature of 'wholeness' sometimes ascribed to 'micro-system/macro-system' composites. Where in HV-3, we would speak of a system consisting of a micro-system and a macro-system we will, in HV-4, speak instead of a physically indivisible 'total-system'. The observables will be analyzed as properties of this 'total-system' rather than as relations between a micro-system and a macro-system. Once again, sensitivity to measurement contexts will be expressed through the states of the total system. HV-4 may be understood in terms of the following axioms:

1) Each system normally described by a quantum state $\emptyset$ will be reanalyzed in terms of the total-system concept and will be described instead by classically determinate total-system states $\lambda_c \in \Lambda_c$. Where $\lambda_{c_i}, \lambda_{c_j} \in \Lambda_c$, $(i \neq j)$, these states are distinct states of a total-system.

2) Corresponding to each quantum observable A there will be a unique random variable $f_A:\Lambda_c \quad R$.

3) With each quantum state $\emptyset$ there will be associated a unique probability measure $\Delta_\emptyset$ on the measurable subsets of Λ_c.

The measure $\Delta_\emptyset$ is subject to the following constraint:

Constraint 4C

a) $P_A^\emptyset(M) = \Delta_\emptyset(f_A^{-1}(M))$

b) $E(A)_\emptyset = \int_{\Lambda_c} f_A(\lambda_c)d\Delta_\emptyset$

Once again, there is no reason to assume that $f_{P_1}(\lambda_{c_i}) = f_{P_1}(\lambda_{c_j})$ $(i \neq j)$, as the Kochen and Specker argument would require.

(d) HV-5: The Many-Measure Approach

In this approach, sensitivity to measurement contexts is expressed through the probability measures employed by the strategy. The existence of a multiplicity of measurement contexts manifests itself theoretically through the existence of a multiplicity of probability measures, one for each context. The strategy may be described in terms of the following axioms:

1) Each system normally described by a quantum state $\emptyset$ will be described instead by classically determinate states $\lambda \in \Lambda$.

2) Each quantum observable A will be associated with a unique random variable $f_A:\Lambda \rightarrow R$.

3) With each quantum state $\emptyset$ there will be associated a family of phase space probability measures $[\Delta_c]$ on the measurable subsets of Λ, one measure [illegible] each context c.

The probabilistic constraint is as follows:

Constraint 5C. Relative to a context c:

a) $P^{\emptyset}_{A}(M) = \Delta_c(f^{-1}(M))$

b) $E(A)_{\emptyset} = \int_{\Lambda} f_A(\lambda)\, d\Delta_c$

In the discussion of the quantum mechanical proofs of the non-existence of joint distributions in Chapter One, it was suggested that one way to avoid the force of these proofs would be to attempt to retrieve the quantum measurement statistics not from a single phase space measure but from a multiplicity of such measures. This is the strategy adopted in HV-5.

SECTION FIVE: COMMENTARY ON THE CONTEXTUAL HIDDEN VARIABLES STRATEGIES

Glymour[36] has the following objection to the approach represented by HV-2: suppose we are measuring a component of a particle's angular momentum in some one direction (say) $\overleftarrow{z}$. Glymour comments:

> This quantity belongs, in ordinary quantum mechanics, to a great many different maximal compatible sets of quantities. But we are not in fact measuring any of these other quantities, so our measuring procedure does not determine any unique maximal compatible set of quantities. But according to the story, what we are measuring depends on the specification of maximal compatible sets of quantities. So we don't know what we are measuring.[37]

In HV-2, just which observable one is measuring (i.e., which element of the family $[f_{Ac}]$, when we are putatively measuring angular momentum along $\overleftarrow{z}$), depends on a specification of the context. Since we typically do not specify the context, then (Glymour concludes) we do not know which observable we are measuring. This pecularity arises from what Glymour calls 'conservation of paradox'. That is, we avoid the Kochen and Specker paradox at the cost of getting embroiled in another paradox.

However, have we really conserved paradox. Arguably we have not. What has happened is that in adopting HV-2 to avoid the Kochen and Specker puzzle we have swapped a mathematical difficulty (due to Kochen and Specker) for an epistemological difficulty (due to Glymour) concerning the identity of observables: we apparently do not know which observable we are measuring when we measure 'angular momentum along $\overleftarrow{z}$.'

Furthermore, while it is true in HV-2 that the observable 'angular momentum along $\overleftarrow{z}$' is associated with a family $[f_{Ac}]$ of random variables, one for each context, we may nevertheless suppose, within HV-2, that the very idea of 'measurement of angular momentum along $\overleftarrow{z}$', without specification of a measurement context, would not constitute a well-defined measurement. Perhaps Glymour's problem could be avoided by a more complete specification of a quantum measurement than the one he considers.

It is at this point that HV-3, HV-4 and HV-5 are relevant. In these approaches there is a one-one correspondence between the quantum observables and the random variables. In HV-5, for instance, what changes from context to context is the distribution of the classically determinate states $\lambda \in \Lambda$, not the observable that is being measured.

In a hidden variables strategy which is sensitive to measurement contexts, the contexts play the role of an additional factor, which, allow the strategy to avoid certain objections (eg., the Kochen and Specker argument) which plague it when contexts are absent. In this regard the contexts are very much like the contractions[38] postulated in FitzGerald-Lorentz aether theory.

Shimony[39] notices, in a manner reminiscent of Glymour, that it is rare for the experimental apparatus to single out a maximal

Boolean sub-algebra (or maximal compatible set of quantities). Shimony, however, remarks of the contexts c of contextual hidden variables strategies:

> Consequently there is a motivation to let c be a 'coarse-grained' state of the environment such as is specified by a macroscopic description of the apparatus . . . Good experimental design in microphysical investigations is, in fact, intended to establish situations in which probabilities are well-defined; situations which exhibit sensitivity to the microscopic features of the system S, sensitivity also to the macroscopic features of the apparatus.[40]

This seems to support the view that considerations of context involve considerations of physically significant factors which are ignored in both ordinary quantum mechanics and in non-contextual hidden variables theories. These factors might well play a role in discussions of what constitutes "good experimental design in microphysical investigations." If this were the case, then Glymour's objection might well be viewed as arising from an incomplete specification of what is going on when we measure 'angular momentum along z.' This is not to dismiss Glymour's objection, but rather to explain what generates it.

To sum up, all hidden variables strategies for quantum mechanics aim to find some way of interpreting quantum mechanics in terms of classically determinate states--states which assign simultaneous exact values to all quantum observables on a system at any time. These values may be measurement-independent observable values or they may simply be values that would be found were certain measurements to be performed. Classical determinacy, then, is what motivates hidden variables strategies.

The non-contextual hidden variables strategies display no sensitivity to the measurements which may be performed on a system. The contextual strategies, on the other hand, consider the many ways in which a given observable can be measured to be important in the construction of a hidden variables strategy. HV-2, HV-3, HV-4 and HV-5 illustrate some of the ways in which 'context sensitivity' can be formally introduced into a hidden variables strategy. The main difference between contextual and non-contextual hidden variables strategies may now be stated: in the non-contextual strategies the aim is to interpret quantum mechanics in terms of classical determinacy by a consideration of quantum mechanical observables alone whereas in a contextual hidden variables strategy the aim is to interpret quantum mechanics in terms of classically determinate states by a consideration of the quantum observables and some new factors (the contexts).

Having discussed a variety of hidden variables strategies I will now present two case studies of hidden variables theories in order to illustrate some of the points made in what has been so far a largely theoretical discussion.

SECTION SIX: CASE STUDY: THE BOHM THEORY

Bohm has worked out in some detail a contextual hidden variables theory of quantum mechanics. It is instructive to examine some of its salient features.

Bohm's original contextual hidden variables interpretation of quantum mechanics appeared in a series of papers in 1952. Bohm[41] has summarized his early work in terms of the following five assumptions:

a) A quantum state $\emptyset$ represents an objectively real field.

b) In addition to the field, there is a particle represented by a set of coordinates which are always well-defined and which vary continuously.

c) The velocity of the particle is given by:

1) $\vec{v} = \frac{\nabla S}{m}$

where m is the mass of the particle and S is a phase function in

2) $Ø = R \exp (i S/h)$,

R and S real.

d) A particle is acted on by a classical potential V(x) and also, in virtue of the Ø-field, by a quantum potential, given by Bohm as:

3) $U = \frac{\hbar^2}{2m} \frac{\nabla^2 R}{R}$

e) The Ø-field is in a state of rapid random fluctuation. According to Bohm, the source of the fluctuations is a sub-quantum domain, with the Schroedinger equation determining the mean behaviour of the Ø-field.

An analogy may be useful here. In some experiments to observe Brownian motion, smoke is captured in a transparent cell. The particles of smoke, when observed through a microscope, appear to be in random (Brownian) motion. The explanation of this behaviour refers to an invisible domain of molecules which collide with particles of smoke. Bohm accounts for quantum fluctuations in an analogous manner: a particle, acted on by the fluctuating Ø-field, will display random continuous motion. His theory has an ontology consisting of classical particles and a random wave field. At first glance, Bohm's theory is reminiscent of the old De Broglie 'pilot wave' theory. But unlike the De Broglie theory, the Bohm theory is readily generalized to cases involving many-particle systems.[42] The Bohm theory is contextual and involves a new variable--the quantum potential. Better understanding of the Bohm theory may be gained by considering its account of the two-slit experiment.

The Two Slit Problem

The two-slit problem--essentially a scattering problem--is schematically illustrated in Figure 2-2. Electrons are imagined to be emitted, one at a time, with a definite momentum, from the electron source equidistant from the two slits in the wall. Those electrons which pass through the slits will have their subsequent positions recorded at the detection screen.

In typical classical calculations[43] for finding a particle at some point x on the screen, given the assumption of mutually exclusive passage at the slits--the electron passes through either S_1 or S_2 but not both--the expression for the probability for arrival at point   is readily seen to be:

1) $P(x) = P(x \text{ and } S_1) + P(x \text{ and } S_2)$

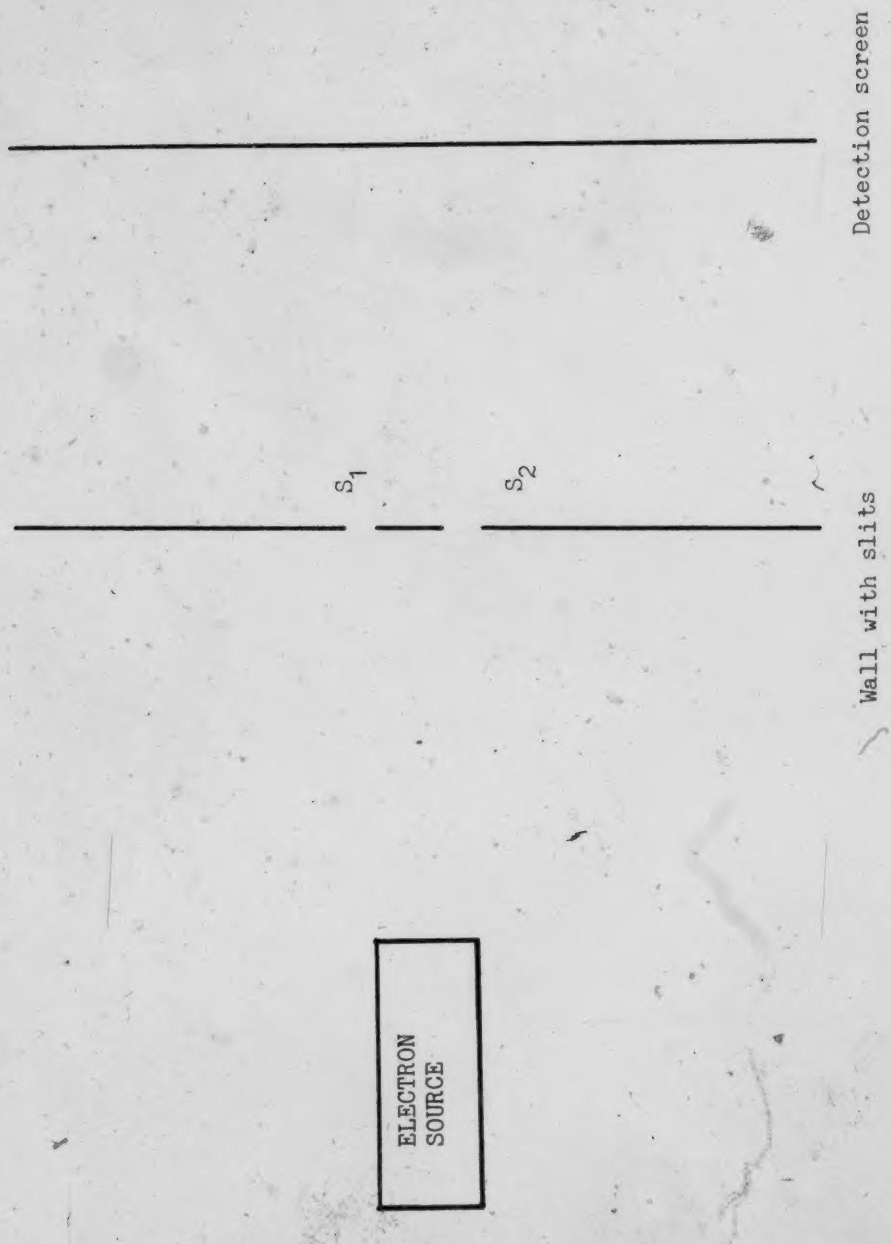
$= P(x/S_1)P(S_1)+P(x/S_2)P(S_2)$

Here 'x' is the event of the electron's arrival at point x, 'S_1' is the event of the electron's passage through slit S_1 etc.

According to quantum mechanics the expression (1) is manifestly inadequate as it suggests that the patterns obtained when S_1 or S_2 are respectively closed should add to yield the pattern obtained when both are open, and not the two-slit diffraction pattern predicted by quantum mechanics. (And suggested by certain experiments).

From the standpoint of orthodox quantum mechanics, let $\emptyset_1(x)$ represent an eigenstate corresponding to 'Electron e passes through

FIGURE 2-2 The Two-Slit Experiment.


Detection screen
S_1
S_2
Wall with slits
ELECTRON SOURCE

S_1'. Let $\emptyset_2(x)$ be an eigenstate corresponding to 'Electron e passes through S_2'. Of those electrons which pass through the wall, $P(S_1) = P(S_2) = 1/2$. If S_1 is open and S_2 is closed:

1) $P_{qm}(x|S_1) = |\emptyset_1(x)|^2$.

If S_1 is closed and S_2 is open:

2) $P_{qm}(x|S_2) = |\emptyset_2(x)|^2$

When both slits are open, the state of an electron which passes through the wall is given by a linear superposition of 'passes through S_1' and 'passes through S_2' states:

3) $\Psi(x) = 1/\sqrt{2}\ \emptyset_1(x) + 1/\sqrt{2}\ \emptyset_2(x)$

In this case, the probability for arrival at x is given by:

4) $P_{qm}(x|S_1 \text{ and } S_2) = |\Psi(x)|^2$

$= |1/\sqrt{2}\ \emptyset_1(x) + 1/\sqrt{2}\ \emptyset_2(x)|^2$

This differs from the sum:

5) $|\emptyset_1(x)|^2 + |\emptyset_2(x)|^2$

by the inclusion of interference terms

6) $\emptyset_1(x)\emptyset_2{}^*(x) + \emptyset_1{}^*(x)\emptyset_2(x)$

This way of calculating probabilities for arrival at points on the screen permits the prediction of the correct two-slit diffraction pattern.

There has been much debate concerning the interpretation of the two-slit diffraction pattern among orthodox theorists.[44] It is sometimes said that when both slits are open, (since the state of the electron is given by the superposition $\Psi(x)$ ((3) above)), the electron passed through neither slit and yet arrived at some point X on the detection screen. On the other hand, it is sometimes said that the electron was 'wavelike' at the slits and so went through both. The subsequent collapse of the wave packet is to explain the particle-like character of the system at the screen. Furthermore, a theorist like Bohr will say nothing at all concerning the electron at the slits since no appropriate 'passed through slit' experiment is performed.

In a number of physical and philosophical discussions of the two-slit experiment, the hypothesis of mutually exclusive passage is held to be suspect. In the Bohm analysis of the two-slit experiment, the hypothesis of mutually exclusive passage is preserved.

In the Bohm theory, the emitted particle is acted on at all times by the quantum potential U, and the probability for arrival at region x on the screen is, as in usual quantum mechanical analysis, given by $|\Psi(x)|^2 dx$. The quantum potential U is affected by macroscopic configurations of the experimental apparatus. In particular, the 0-field which gives rise to the quantum potential U is deformed differently by a wall with two slits in it than by a wall with one slit in it. Thus in the Bohm theory the quantum measurement statistics are retrieved for the two-slit experiment by maintaining the hypothesis of mutually exclusive passage and using an ontology of classically determinate particles with continuous trajectories. The explanation of the various diffraction patterns makes essential use of the quantum potential U and the way it is altered by distinct configurations of the apparatus. As Bohm put it:

> . . . the distribution of hidden parameters varies in accordance with the different mutually exclusive experimental arrangements of matter that must be used in making different kinds of measurements. In this point we are in agreement with Bohr, who repeatedly stresses the fundamental role of the measuring apparatus as an inseparable part of the observed system.[45]

In the above quotation it can be seen that the Bohm theory is sensitive to the various measurement contexts--mutually exclusive experimental arrangements--that are possible. What changes from context to context is the distribution of the hidden parameters or classically determinate states. This is precisely the way in which sensitivity to measurement contexts is introduced in HV-5. It is reasonable to suppose that the Bohm theory is a realization of the HV-5 strategy. There is a sense, of course, in which the Bohm theory goes beyond HV-5. In the Bohm theory it is not merely the case that all quantum observables take simultaneous exact values (classical determinacy). In this theory there is a new observable--the quantum potential--which is the counterpart in the Bohm theory of the contractions in FitzGerald-Lorentz aether theory.

SECTION SEVEN: CASE STUDY: STOCHASTIC ELECTRODYNAMICS

The theory of stochastic electrodynamics has been the subject of intensive investigation over the last twenty or so years, as Boyer's[46] bibliography shows. The version of stochastic electrodynamics to be considered here is due to Boyer.[47]

Stochastic electrodynamics is an electrodynamical theory which aims to recover a range of quantum measurement statistics, giving the phenomena underlying these statistics an electrodynamical interpretation.

In stochastic electrodynamics, a dualistic ontology of classical waves and particles is postulated--reflecting the view that stochastic electrodynamics belongs to the class of classical electrodynamical theories. A classical electrodynamical theory can be understood as consisting of the following items:

a) Newton's equations of motion for particles.

b) Maxwell's equations for the electromagnetic field.

c) Boundary conditions for (a) and (b) above.

Different choices of boundary conditions generate different classical electrodynamical theories.

Important for the present purposes is the difference between traditional electrodynamics[48] and stochastic electrodynamics. In both of these theories the solution of Maxwell's equations can be seen as the sum of the transverse solutions of the homogeneous and inhomogeneous vector wave equations. Thus:

a) $\vec{A}(\vec{r},t) = \vec{A}_0(\vec{r},t) + \vec{A}_s(\vec{r},\ t)$,

for $\vec{A}$ a vector potential, $\vec{A}_0(\vec{r},t)$ the solution of the homogeneous vector wave equation and $\vec{A}_s(\vec{r},t)$ the solution of the inhomogeneous vector wave equation.

In traditional electrodynamics,

2) $\vec{A}_0(\vec{r},t) = 0$

and

3) $\vec{A}_s(\vec{r},t) = \int \vec{J}_\perp(\vec{r},t)G^r(\vec{r},t;\ \vec{r}',t')d^4x'$,

where $\vec{J}$ is the current density of the source of the radiation and G^r is the retarded Green function:

4) $G^r(\vec{r},t;\ \vec{r}',t') = G(\vec{r},t;\ \vec{r}',t')$ for $t > t'$

$= 0$ for $t < t'$

Two important boundary conditions in traditional electrodynamics are use of the retarded Green function and the setting of the solution to the homogeneous vector wave equation to zero. These boundary conditions reflect the traditional view that all electromagnetic radiation is traceable back to sources (accelerating charges) and that in the absence of appropriate sources, there is no radiation.

Stochastic electrodynamics employs the retarded Green function, but now $\vec{A}_0(\vec{r},t) \neq 0$. As Boyer puts it:

> This theory assumes that the homogeneous solution to Maxwell's equations involves random classical radiation . . . The aspect of randomness in the theory involves the averaging over many microscopic but deterministic degrees of freedom. The theory describes accurately a number of phenomena usually thought to require a quantum description.[49]

This random classical radiation is postulated to have the properties of homogeneity, isotropy, Lorentz invariance and temperature independence (it is a zero point field). It is standardly claimed that the random radiation field has an average energy density of 1/2 $\hbar\omega$ per normal mode of angular frequency ω. The spectrum is given by:

5) $\rho(\omega) = (\omega^2/\pi^2c^3)1/2\hbar$

The assumption that the normal mode energy density is $1/2\hbar\omega$ implies that Planck's constant (divided by 2π) determines the magnitude of the field fluctuations. It is at this point, and at this point only, that Planck's constant enters stochastic electrodynamics, and it enters the theory in a manner not suggestive of quantization of action.

The hidden variables strategist has a number of reasons for being interested in stochastic electrodynamics. First, there is the treatment of particles as classical particle systems. Inasmuch as the theory can explain quantum phenomena, these particle-systems will take values for quantum observables (such as 'spin'). There is thus the possibility of classical determinacy as regards the observables which quantum mechanics ascribes to systems. Secondly, there is the absolute ontological distinction between classical waves and classical particles.[50] There are thus no ontological chameleons (or worse) working in the theory. Thirdly, stochastic electrodynamics employs classical techniques of statistical analysis. Essential use is made of the random variable/phase space apparatus and the randomness of the zero point field is seen as the result of averaging over many deterministic degrees of freedom. Fourthly, Nelson has considered the motion of a classical particle subject to a random field and has produced a 'classical' derivation of the Schroedinger equation.[52] In stochastic electrodynamics it has been hoped that by consideration of the classical motion of a particle subject to random electromagnetic radiation, the Schroedinger equation may be derived as a classical stochastic equation of motion. Success has not been unambiguous in this regard.[53] Fifthly, and perhaps most important for the present purposes, is the potential within stochastic electrodynamics for a classical account of spin-measurement statistics.[54]

Stochastic electrodynamics may be summarized as an electrodynamical version of a De Broglie-type (i.e., 'pilot wave') hidden variables strategy. The successes and failures of stochastic electrodynamics, in terms of hard calculations, have been reviewed by Boyer[55] and De La Pena and Cetto.[56] What matters for the present purposes is the contextual nature of the stochastic electrodynamical strategy.

Consider the two-slit experiment. According to Boyer,[57] the spectrum of fluctuations in zero point radiation is modified by the presence of matter which interacts with the radiation field. In terms of the two-slit experiment there is a real difference in the fluctuation pattern of the zero point radiation when one slit is open rather than two. Considering the case in which both slits are open, Boyer speculates:

> . . . in our classical view, the particle passes through only one slit, but the pattern of zero-point fluctuations reflects the presence of both slits. Covering one of the slits of course changes the radiation fluctuation pattern, and accordingly changes the influence on the passing particle.[58]

Just as in the Bohm theory, field effects are exploited in order to preserve the hypothesis of mutually exclusive passage in the face of the two-slit diffraction pattern. Whereas Bohm postulates a random 0-field Boyer postulates a random zero point field of classical electromagnetic radiation.

Boyer's handling of the two-slit problem is illustrative of the basic methodology of stochastic electrodynamical explanation: where there is a phenomenon which quantum mechanics predicts but which is not explicable within traditional electrodynamics, then one finds an explanation of that phenomenon within stochastic electrodynamics by exploiting the zero point field effects (which are absent from traditional electrodynamics).

This methodology has been used successfully by Boyer in terms of analytical calculations for black body radiation, atomic stability, van der Waals forces and a number of oscillator systems. It has also been used informally to explain the effects of the measuring apparatus on the system being measured. Boyer comments:

> This notion of the unavoidable influence of the measuring apparatus is a natural deduction in our classical theory with zero point radiation. The measuring apparatus will involve matter with electromagnetic interactions. This matter changes the pattern of zero point radiation near the apparatus and so alters the system being observed.[59]

The Boyer theory is sensitive to the different measurements that may be performed on a system in that the measuring apparatus, through its effects on the zero point field, makes a contribution to the resulting observable-value for the observable being measured. There is a clear sense in which stochastic electrodynamics is a contextual theory.[60]

FOOTNOTES

[1] Cartwright (1983), p. 168.

[2] Shanks (1985). This paper reviews some of the issues in this Chapter.

[3] E.g., Jammer (1974), pp. 261-265.

[4] '$Val(A)_0 \in M$' is to be read as 'Observable A in state 0 has a value lying in Borel set M.

[5] Kochen and Specker (1967).

[6] Von Neumann (1955), Ch. 4, Section One and Section Two.

[7] Jammer (1974), pp. 267-268.

[8] Ibid., pp. 265-278.

[9] Bell (1966).

[10] This is certainly the orthodox view concerning the significance of the non-cummutativity of observables A and B.

[11] van Fraassen (1980), p. 53.

[12] Ibid.

[13] Kochen and Specker (1967).

[14] Ibid., p. 64. Considering a three-dimensional vector space, Kochen and Specker define a partial algebra as follows: A set 0 forms a partial algebra over a field K if there is a binary relation R called pairwise commutativity on 0--$R \subseteq 0 \times 0$--with the operations of addition and multiplication from R to 0, scalar multiplication from $K \times 0$ to 0, and an element 1 of 0 satisfying the following:

1) R is reflexive and symmetric: aRa for all $a \in 0$, and aRb implies bRa for all $a,b \in 0$.

2) For all $a \in 0$, aR1.

3) The relation R is closed under the operations: if a_iRa_j for all $1 \leq i$; $j \leq 3$ then $(a_1 + a_2)Ra_3$, $a_1a_2Ra_3$ and ka_1Ra_3, for all $k \in K$.

4) If a_iRa_j for all $i \leq 1$, $j \leq 3$, then the values of the polynomials in a_1, a_2, a_3 form a commutative algebra over the field K.

Kochen and Specker point out that every commutative algebra C forms a partial algebra if R is taken to be CxC. The set of idempotents of a commutative algebra form a Boolean algebra and the set of idempotents of a partial algebra form a partial Boolean algebra.

[15] $V_3(R)$ is a three-dimensional Hilbert space over the field of reals.

[16] This matter will receive further attention in Chapter six of this study.

[17] Kochen and Specker (1967), p. 63.

[18] Ibid., p. 63.

[19] For a further detailed analysis of (K and S) the reader should consult Sharp (1978), Ch. 9.

[20] Sharp (1978).

[21] See Ballentine (1970) and Audi (1974) for details concerning the statistical interpretation of quantum mechanics.

[22] Ballentine (1970).

[23] Ibid., p. 361.

[24] Sharp (1978), p. 252.

[25] Fine (1974), p. 266.

[26] If quantum mechanics does not force the view that $Val(g(A)) = g(Val(A))$ then it does provide inductive support for this relation between the values of A and g(A). If we look at systems in $E(\psi_i)$ it is quantum mechanically the case that the probability that a system possesses a for A is the same as that it possesses g(a) for g(A).

[27] Sharp (1978), p. 247.

[28] Ibid., pp. 298-299.

[29] Ibid., p. 378.

[30] Bohr (quoted in Bell (1966)).

[31] P_1 corresponds to $Ø_1$, etc.

[32] Bell (1966), pp. 450-451. See also Bell (1982).

[33] **A** and **B** do not share an orthonormal basis in $V_3(R)$.

[34] Bell (1982).

[35] Shimony (1984), p. 35.

[36] Glymour (1976).

[37] Ibid., p. 165.

[38] See Chapter One, Section One and Section Two of this study.

[39] Shimony (1984).

[40] Ibid., pp. 29-30.

[41] Bohm (1980), pp. 76-80. The reader should also consult Bohm (1952) for precise details.

[42] See Bohm (1952); Bohm and Hiley (1982).

[43] See Fine (1972), pp. 5-6; Cartwright (1983), pp. 175-176.

[44] E.g., Fine (1972), pp. 23-27; Merzbacher (1961), pp. 9-12.

[45] Bohm (1952), pp. 187-188.

[46] Boyer (1980).

[47] Boyer (1975).

[48] E.G., Lorentz (1908).

[49] Boyer (1975), p. 792.

[50] There is no second quantization in stochastic electrodynamics.

[51] Nelson (1968).

[52] See Jammer (1974), Ch. 9.

[53] See Boyer (1980).

[54] De La Pena and Cetto (1982); Sachindanandan (1983).

[55] Boyer (1980).

[56] De La Pena and Cetto (1982).

[57] Boyer (1975).

[58] Ibid., p. 803.

[59] Ibid., p. 803.

[60] Stochastic electrodynamics has a number of defects and shortcomings. See Boyer (1980) and De La Pena and Cetto (1982). As to how serious these will ultimately turn out to be is not clear at the time of writing.

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CHAPTER THREE

BELL'S THEOREM

> The paradox of Einstein, Podolsky and Rosen was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality.
>
> J.S. Bell[1]

SECTION ONE: INTRODUCTION

In this Chapter the Einstein, Podolsky and Rosen paradox (EPR), and Bell's Theorem will be presented. Bell's theorem will be examined from both a theoretical and an experimental perspective, and its implications for hidden variables strategies and the issue of determinism versus indeterminism will be analyzed.

SECTION TWO: THE EPR PARADOX

In 1935[2] Einstein, Podolsky, and Rosen argued that quantum mechanics does not provide a complete analysis of physical systems in the sense that there are apparently properties of systems in the domain of quantum mechanics which are not represented within the theory. Their argument rests on four assumptions:[3]

1) The statistical predictions of quantum mechanics, insofar as they are relevant to the argument, are correct.

2) Einstein Locality: There is no action-at-a-distance or superluminary action. In particular, if two systems that previously interacted are spacelike separated, then no real change can take place

immediately in one system as the result of anything which may be done to the other system.

3) The Reality Criterion: If, without disturbing a system, it is possible to predict with probability one the value of an observable, then that observable has that value independently of measurement.

4) The Completeness Criterion: A physical theory is complete iff every observable which has a value independently of measurement has this value represented within the theory.

Bohm[4] has presented a version of the EPR argument which concerns the spin observable. It is this version of the EPR paradox which Bell discusses.[5]

The system under analysis is a pair of spacelike separated spin-1/2 particles (1 + 2) prepared in the singlet spin state Ψ_0:

1) $\Psi_0 = 1/\sqrt{2}\,[\phi_1(+1)\phi_2(-1)-\phi_1(-1)\phi_2(+1)]$

According to quantum mechanics, when spin along some arbitrary direction $\vec{a}$ is measured on either particle, then Ψ_0 'collapses' and, depending on the result of measurement (by the Projection Postulate), reduces to either:

2) $\eta_1 = \phi_1(+1)\phi_2(-1)$,

or

3) $\eta_2 = \phi_1(-1)\phi_2(+1)$,

where η_1 corresponds to spin +1 along $\vec{a}$ on particle 1 and spin -1 along $\vec{a}$ on particle 2. Similarly for η_2. Thus quantum mechanics predicts that if spin is measured along some direction $\vec{a}$ on one particle then the value of spin along that direction $\vec{a}$ on the other particle will be found to be opposite to the measured value on the original particle. This is the anti-correlation feature of quantum spin component measurement statistics. Thus, we can predict with probability one the result of measuring any chosen component of spin on one particle after previously measuring the same component of spin on the other.

By hypothesis, the particles of interest are spacelike separated--and they have separated freely from some common source. By Einstein Locality, then, no change can occur in one particle as the immediate result of measurements done on the other (indeed, the particles could be light years apart). Now since we can predict with certainty the result of measuring a spin component on one particle immediately after measuring the same spin component on the other, and since by Einstein Locality, the result of measuring a spin component on one particle cannot be immediately affected by measurements done on the other, then by the Reality Criterion there must be measurement-independent spin component values for all spin components.[6]

The singlet state ψ_0 does not determine the result of any individual measurement. So by the Completeness Criterion there are measurement-independent values for spin components which are not represented within quantum mechanics. Therefore quantum mechanics is incomplete.

The Bohrian rejoinder to this argument concerns the Reality Criterion used by Einstein, Podolsky and Rosen. We saw in Chapter One that Bohr believed that in quantum physics, unlike classical physics, the micro-object and the measuring apparatus form an indivisible totality which cannot, properly speaking, be broken down into parts, one of which is the micro-object with definite values for all spin components. Consequently, Bohr would claim that if we were to measure a spin component on a particle and find a value, no inference can be made to the particle's having had that value immediately prior to, or independently of, measurement. Bohr believed, in essence, that the Reality Criterion required an unwarranted 'divisibility' of events. He commented:

> The unambiguous account of proper quantum phenomena must, in principle, include a description of all relevant features of the experimental arrangement . . . In the case of quantum phenomena, the unlimited divisibility of events implied by such an account is, in principle, excluded by the requirement to specify the experimental conditions. Indeed, the feature of wholeness typical of proper quantum phenomena finds its logical expression in the circumstance that any attempt at a well-defined subdivision would demand a change in the experimental arrangement incompatible with the definition of the phenomena under investigation.[7]

According to Bohr, quantum measurements do not, *pace* Einstein, Podolsky and Rosen, reveal information about particles or other physical systems under analysis, either directly or indirectly by inference. Quantum measurements, on this view, give results which pertain to the entire experimental arrangement.

According to the Bohrian view, quantum mechanics is a theory which has as its main concern measurement results and their statistics (ie., empirical and predictive adequacy). Some aspects of Bohr's philosophy will be returned to in Chapter Five, where van Fraassen's views are discussed. Should Bell's theorem turn out to be a convincing 'no hidden variables' argument, then one must remember that Bohr's philosophy is always waiting in the wings for an interpretative curtain call.

The important point to be borne in mind at this juncture is that notwithstanding the views of Bohr, some theorists have wondered whether a more complete, i.e., classically determinate, description is possible of the puzzling particle-pairs in the singlet spin state. This is the concern of Bell's argument.[8]

SECTION THREE: THE BELL ARGUMENT

In quantum mechanics, the expectation value of the product of two spin components $S_1(\vec{a})$ and $S_2(\vec{b})$[9] in the singlet state ψ_o is:

$$1)\quad E(S_1(\vec{a})S_2(\vec{b}))_{\psi_o} = -\vec{a}\cdot\vec{b} = -\cos\theta_{ab}$$

where θ_{ab} is the angle between $\vec{a}$ and $\vec{b}$. Bell wonders whether one could construct an interpretation of quantum mechanics in which the results of measuring spin components are classically determinate--that is, can one interpret quantum mechanics in terms of states λ which are lists of values that would be found were spin component measurements to be performed.[10]

This is, of course, just a version of the hidden variables question in quantum mechanics. To fulfill this project we require a classical probability space:

$$2)\quad \Omega = \langle \Lambda, \sigma(\Lambda), P \rangle$$

The classically determinate states $\lambda \in \Lambda$ are, as far as the Bell argument is concerned, just lists of spin component values that would be found were appropriate measurements to be performed. All spin component values on the particle-pairs are listed. Bell does not make the assumption that spin components take values independently of measurement; he is neutral on this matter.

With respect to the classical probability space Ω, Bell analyzes the measurement results $A(\vec{a},\lambda)$ and $B(\vec{b},\lambda)$[11] as random variables on Λ. $A(\vec{a},\lambda) = \pm 1$, $B(\vec{b},\lambda) = \pm 1$. Thus the measurement result A depends only on the direction $\vec{a}$ along which spin is measured, and the classically determinate state $\lambda \in \Lambda$. Similarly for the result B.

Bell argues that it is reasonable to assume that non-local influences must be ruled out in the construction of a hidden variables interpretation of quantum mechanics--the interpretation must be Lorentz invariant, (i.e., it must be Einstein local). How is the condition of Einstein locality to be represented in the hidden variables model for the quantum spin correlation measurement statistics? Bell's answer is through the constraint of Bell Locality.

3) Bell Locality $=_{df}$ (for spacelike separated particles)

(a) The result $A(\vec{a},\lambda)$ is to be independent of the direction $\vec{b}$ along which spin is measured on particle 2, and similarly for the result $B(\vec{b},\lambda)$.

(b) The correlation statistics must be derived from a common phase space probability density ρ, where

$$\int_{\Lambda} \rho(\lambda)d\lambda = 1$$

An important question is whether Einstein locality always implies Bell locality. This question will be taken up again in Chapter Five. What will be established here is the following claim: Einstein locality implies Bell locality only if the state of the measuring apparatus (the setting of the Stern-Gerlach magnets used to measure components of spin) is independent of the states λ of the emitted particle-pairs. This is the principle of *system-apparatus independence*.

We know that Einstein locality requires that the result $A(\vec{a},\lambda)$ be independent of the direction $\vec{b}$ along which spin is measured on particle 2 (and similarly for the result $B(\vec{b},\lambda)$). The assumption of system-apparatus independence means that there is no non-arbitrary way of introducing a multiplicity of phase space probability densities, one for each distinct $\vec{a},\vec{b}$-experiment. On the assumption that there is no conspiracy between the emitted systems and the measuring apparatus--i.e., system-apparatus independence--then the probability density ρ will be a function of λ alone, just as Bell locality requires. If the states of the apparatus are not independent of the states of the emitted systems then Einstein locality will not imply Bell locality. We may now see the constraint of Bell locality for what it is: *it is a constraint which forbids the introduction into a hidden variables theory of sensitivity to $\vec{a},\vec{b}$-measurement contexts*. And of course, if systems were emitted from the source in certain states λ only if certain experiments were to be subsequently performed on them[12] then $\rho = \rho(\vec{a},\vec{b},\lambda)$, for instance, and Bell locality would be violated even though there was no violation of Einstein locality. The point to be emphasized here--and it will be important in Chapter Five--is that a hidden variables theory can be sensitive to $\vec{a},\vec{b}$-measurement contexts without this necessarily meaning that it violates Einstein locality.

However, granting Bell's assumption of hidden variables $\lambda \in \Lambda$, and granting the assumption of Bell locality, the correlation statistics will be calculated as follows:

4) $$E(\vec{a},\vec{b}) = \int_{\Lambda} A(\vec{a},\lambda)B(\vec{b},\lambda)\rho(\lambda)d\lambda = -\cos\theta_{ab}$$

The following derivation of Bell's theorem is due to Bell.[13]

The quantum mechanical account of the singlet state ψ_0 requires that

5) $$A(\vec{b},\lambda) = -B(\vec{b},\lambda)$$

Bell locality (clause (a)) requires that the result of measuring spin along b on particle 2 be independent of the direction along which spin is measured on particle 1, and similarly for particle 1. Bell locality plus the quantum prediction (5) allows us to measure the spin along any direction on particle 2, without in any way disturbing particle 2, by previously measuring the same component of spin on particle 1. Given this, (4) becomes

6) $$\int_{\Lambda} A(\vec{a},\lambda)A(\vec{b},\lambda)\rho(\lambda)d\lambda = \cos\theta_{ab}$$

That is, Bell locality plus the quantum prediction (5) allows us to drop any further reference to particle 2. The rest of the argument will now concern spin component measurement results on particle 1.

Using (6) we get:

7) $$\cos\theta_{ab} - \cos\theta_{ac} = \int_{\Lambda} [A(\vec{a},\lambda)A(\vec{b},\lambda)-A(\vec{a},\lambda)A(\vec{c},\lambda)]\rho(\lambda)d\lambda$$

Since $A(\vec{b},\lambda) = \pm 1$, $[A(\vec{b},\lambda)]^2 = 1$, so (7) becomes:

8) $$\cos\theta_{ab} - \cos\theta_{ac} = \int_{\Lambda} A(\vec{a},\lambda)A(\vec{b},\lambda)[1-A(\vec{c},\lambda)A(\vec{b},\lambda)]\rho(\lambda)d\lambda$$

Since $A(\vec{a},\lambda)A(\vec{b},\lambda) \leq 1$, we may use (8) to get

$$9)\quad \cos\theta_{ab} - \cos\theta_{ac}$$

$$\leq \int_{\Lambda} [1 - A(\vec{c},\lambda)A(\vec{b},\lambda)]\rho(\lambda)d\lambda$$

By (6) and the fact that:

$$10)\quad \int_{\Lambda} \rho(\lambda)d\lambda = 1,$$

it follows that the right hand side of the inequality (9) is $1 - \cos\theta_{bc}$. Rearranging terms in (9) gives:

11) $\cos\theta_{ab} + \cos\theta_{bc} \leq 1 + \cos\theta_{ac}$ (Bell's Inequality)

The inequality (11) cannot be satisfied in general since, in the case where $\vec{a}$ and $\vec{c}$ are orthogonal and $\vec{b}$ bisects them, $\theta_{ab} = \theta_{bc} = \pi/4$ and $\theta_{ac} = \pi/2$, with cosines $1/\sqrt{2}$ and 0 respectively. If this is the case the inequality (11) yields the contradiction $\sqrt{2} \leq 1$ (Bell's Theorem). Therefore, we must either drop the assumption of Bell locality or the assumption of hidden variables (Bell's dilemma).

Failure to distinguish Bell locality from Einstein locality has led to the widespread view that the Bell argument is a 'no hidden variables' proof. Indeed, it is rare to find a theorist who distinguishes between Bell locality and Einstein locality.[14] As regards the hidden variables issue, Bell himself[15] is neutral. He leaves us with a dilemma..

Some comments are in order at this point. Bell's theorem establishes that there can be no Bell local hidden variables strategy. Non-contextual hidden variables strategies are not sensitive to measurement context. It is very natural for such approaches to take the probability density ρ to be a function of λ alone (i.e., to satisfy clause (b) of Bell locality). If such strategies are to avoid the force of the Bell argument they must violate clause (a) of Bell locality. That is, the result $A(\vec{a},\lambda)$ must not be independent of the direction $\vec{b}$ along which spin is measured on particle 2, and similarly for the result $B(\vec{b},\lambda)$. Since the direction $\vec{b}$ need not be selected until such a time as luminal or sub-luminal communication between the spacelike separated Stern-Gerlach magnets is impossible, such hidden variables strategies would appear to end up on the shoals of Einstein non-locality.

Bell's theorem rules out any Bell local contextual theory. Contextual hidden variables theories are so-called because of their sensitivity to measurement contexts. Contextual hidden variables theories that are sensitive to the $\vec{a},\vec{b}$-measurement contexts will certainly not be *Bell* local and so they will not be ruled out by Bell's theorem. It is a good question, however, as to whether, in virtue of not being Bell local, such theories must necessarily be *Einstein* non-local. This question will be dealt with fully in Chapter Five. The point to be stressed here is that the question is by no means a trivial one.[16] The principle of system-apparatus independence is needed to get from Einstein locality to Bell locality. It is conceivable that this principle could fail to be satisfied.

Secondly, while Bell's original argument concerned λ which were nothing more than lists of values that would be found were certain experiments to be performed,[17] it is nevertheless clear that the argument would apply with the same force to hidden variables theories whose λ were nothing more than lists of measurement-independent spin component values.[18] It is because of this latter point that one might be tempted to say that Bell has shown that Einstein's desire for a complete quantum mechanics could only be satisfied in a way Einstein would have liked least (via the introduction of non-local effects). One can say this if one believes that Einstein locality implies Bell locality--which it does on the assumption of system-apparatus independence. If this assumption fails then Einstein might yet have his cake and eat it.

A third point to be emphasized is this: Bell's argument is not a dynamical argument. It concerns a pair of spin-1/2 particles in the singlet spin state at some fixed time t. It does not concern systems over time. Hence the argument is concerned with classical determinacy versus classical indeterminacy. The argument is relevant to the determinist/indeterminist debate only insofar as certain conceptions of determinism (and indeed indeterminism) require classical determinacy. This point was recognized by Bell who commented:

> In a complete theory of the type envisaged by Einstein, the hidden variables would have dynamical significance and laws of motion; our λ can be thought of as initial values of these variables at some suitable instant.[19]

The final point I want to make here is this: whereas there has been a general tendency among commentators to regard the Bell argument as a no hidden variables proof, one might bite the bullet of Bell non-locality and hence be a hidden variables theorist in the face of the Bell argument. Indeed, some hidden variables theories are not merely not Bell local, they are avowedly not Einstein local either. Bohm's theory of 1952 is interesting in this regard. As we saw in the last Chapter, that theory was a contextual hidden variables theory. The theory is explicitly not Einstein local, let alone Bell local. Bohm and Hiley[20] note that in the 1952 Bohm theory, the hidden variables are the actual positions X of all the particles constituting the total system of observed object plus all pieces of measuring apparatus. The idea is to introduce a wave function $\Psi(X_i,t)$ which satisfies the Schroedinger equation for the corresponding many-body problem. Bohm and Hiley make the following observations concerning this theory:

> For the case of spin measurements, the total wave function will be functionally related to the orientations of both pieces of observing apparatus, so that we can write
>
> $$\Psi = \Psi(\vec{a},\vec{b},X_i,t) = \Psi(\vec{a},\vec{b},\lambda,t)$$
>
> (where we have denoted the set of hidden variables X_i by the symbol λ). We now assume that in addition to the classical potential operating on the hidden variables, there is a further quantum potential.
>
> $$Q = Q(\Psi(\vec{a},\vec{b},\lambda,t))$$
>
> where Q is a certain specified function of the overall wave function. We then show that on the basis of this potential it is consistent to define a probability distribution over λ, which is
>
> $$\rho = \rho(\Psi(\vec{a},\vec{b},\lambda,t))$$
>
> In this model, the observed result for each piece of apparatus corresponds to a set of hidden variables associated with that piece of apparatus alone . . . However, as indicated above, Bell's second condition $\rho = \rho(\lambda)$ is not satisfied, so that Bell's inequality is violated. In this case the dynamical hidden variables of the system are non-local in the sense that the quantum potential, Q, permits strong instantaneous interactions of particles at indefinitely large distances.[21]

Bohm and Hiley are right that the Bohm theory is not Einstein local--the quantum potential Q reflects the entire $\vec{a},\vec{b}$-measurement context. Bell locality is violated in that in the Bohm theory $\rho \neq \rho(\lambda)$.

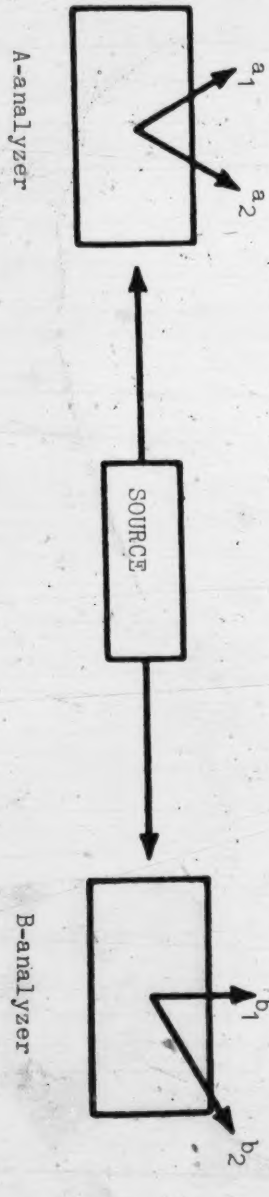
SECTION FOUR: AN EXPERIMENTAL VERSION OF BELL'S ARGUMENT

There are many extant versions of the Bell argument. The one to be discussed below examines the Bell argument from an experimental perspective. As we saw in the last section, crucial to Bell's proof of his theorem is a derivation of the Bell inequality. In the case to be discussed here, Bell's inequality reduces to the Clause-Horne inequality (CH). As will be seen, this inequality constitutes a formal constraint on the random variable/phase space apparatus from which the hidden variables theorist hopes to recover the quantum spin correlation measurement statistics.

Given the assumption of Bell locality, Clauser and Horne[22] derive in a straightforward fashion a Bell-type inequality which would have to be satisfied by any Bell local hidden variables interpretation of the quantum spin correlation measurement statistics for spin components along four directions ($\vec{a}_1$, $\vec{a}_2$ for particle 1 and $\vec{b}_1$, $\vec{b}_2$ for particle 2). A version of Fine's[23] proof of the Clauser-Horne inequality was recently presented by Sharp and Shanks.[24] Details of that proof are reproduced below.

In Figure 3-1 there is a schematic representation of the kind of spin correlation experiment under consideration.[25] Particles are emitted from the source in pairs, each pair prepared in the singlet spin state. In any pair of particles, particle 1 will be supposed to travel into the A-wing (the wing containing the A-analyzer) and particle 2 will be supposed to travel into the B-wing. The A-analyzer and the B-analyzer will be supposed to be Stern-Gerlach magnets with an appropriate arrangement of detectors. Each magnet has, for any given experiment, two orientations (as in Figure 3-1).

FIGURE 3-1 Quantum Correlation Experiments.


A-analyzer
a_1
a_2
SOURCE
B-analyzer
b_1
b_2

Assume there are hidden variables $\lambda \in \Lambda$. Each λ can be thought of as nothing more than a list of values for the response functions which represent the observables of the experiment. Response functions (i.e., random variables) A_i and B_j are defined as follows:

1) $A_i : \Lambda \rightarrow [0,1]$

(Similarly for B_j). The random variable A_i is to be understood as follows:

2) $A_i(\lambda) = 1$ iff particle 1 in a pair (1+2) in classically determinate state λ will pass a barrier indicating spin +1 along the direction $\vec{a}_i$. ($B_j(\lambda) = 1$ iff particle 2 in a pair (1+2) in classically determinate state λ will pass a barrier indicating spin +1 along $\vec{b}_j$). Furthermore:

3) $\overline{A}_i(\lambda) = 1$ iff $A_i(\lambda) = 0$

(Similarly for $\overline{B}_j(\lambda)$).

In the experimental setting under consideration, where particle-pairs are in classically determinate states λ which determine their responses to orientations of analyzers, we thus have eight random variables (two random variables for each of the four directions under analysis). The probability density function $\rho(\lambda)$ will induce probabilities for various individual responses. These probabilities may be understood as follows:

1) $$P(A_i) = \int_{\Lambda_{A_i}} \rho(\lambda)d\lambda$$

where $\Lambda_{A_i} = [\lambda : \lambda \in \Lambda$ and $A_i(\lambda) = 1]$. Thus $P(A_i)$ is the probability of finding the response spin +1 along the direction $\vec{a}_i$

for particle 1 in a pair (1+2). $P(B_j)$ is to be understood in an analogous fashion. Probabilities will also be induced for various joint responses such as:

2) $$P(A_i\bar{B}_j) = \int_{\Lambda_{A_i\bar{B}_j}} \rho(\lambda)d\lambda,$$

where $\Lambda_{A_i\bar{B}_j} = [\lambda:\lambda \in \Lambda$ and $A_i(\lambda) = \bar{B}_j(\lambda) = 1]$. Thus $P(A_i\bar{B}_j)$ is the probability of finding the response spin +1 along $\vec{a}_i$ for particle 1 and the response spin -1 along $\vec{b}_j$ for particle 2 in a pair (1+2). Probabilities for joint responses $P(A_1A_2B_1B_2)$ are similarly defined. Indeed, we have a complete set of probabilities for eight outcomes and combinations thereof (as indeed we must, since the random variables are defined on a common phase space). The Clauser-Horne inequality is a straightforward consequence of the fact that these probabilities must bear certain relations to one another. Thus we have:

1) $P(\bar{A}_1A_2B_1) \leq P(\bar{A}_1B_1) = P(B_1)-P(A_1B_1)$

and

2) $P(A_1A_2B_1) \leq P(A_1A_2)$.

Since

3) $P(A_2B_1) = P(\bar{A}_1A_2B_1) + P(A_1A_2B_1)$,

addition of (1) and (2) yields:

4) $P(A_2B_1) + P(A_1B_1) - P(B_1) \leq P(A_1A_2)$.

Analogous to (2), we have

5) $P(\bar{A}_1A_2B_2) \leq P(\bar{A}_1A_2)$

and

6) $P(A_1A_2B_2) \leq P(A_1B_2)$

Adding these and using an obvious analogue of (3), we have:

7) $P(A_2B_2) - P(A_1B_2) \leq P(\overline{A}_1A_2)$

Now

8) $P(A_2) = P(A_1A_2) + P(\overline{A}_1A_2)$

By adding (4) and (7) and using (8), we then get:

9) $P(A_1B_1) + P(A_2B_1) + P(A_2B_2)$

$- P(A_1B_2) - P(A_2) - P(B_1) \leq 0$

Again, analogous to (2), we have:

10) $P(A_1A_2B_2) \leq P(A_2B_2)$

and

11) $P(A_1A_2\overline{B}_2) \leq P(A_1\overline{B}_2) = P(A_1) - P(A_1B_2)$

Again, by adding and using an obvious analogue of (3), we obtain:

12) $P(A_1A_2) \leq P(A_2B_2) + P(A_1) - P(A_1B_2)$

Moreover,

13) $P(\overline{A}_1A_2B_1) \leq P(A_2B_1)$

and

14) $P(\overline{A}_1A_2\overline{B}_1) \leq P(\overline{A}_1\overline{B}_1)$

$= P(\overline{A}_1) - P(\overline{A}_1B_1)$

$= P(\overline{A}_1) - P(B_1) + P(A_1B_1)$

$= 1 - P(A_1) - P(B_1) + P(A_1B_1)$

Once more, adding and using an obvious analogue of (3), we have:

15) $P(\overline{A}_1A_2) \leq P(A_2B_1) + 1 - P(A_1)$

$- P(B_1) + P(A_1B_1)$

Adding (12) and (15) and using the fact that $P(\overline{A}_1A_2) +$ $P(A_1A_2) = P(A_2)$, we get

16) $-1 \leq P(A_1B_1) + P(A_2B_1) + P(A_2B_2)$

$- P(A_1B_2) - P(A_2) - P(B_1)$

Combining (9) and (16) we get the Clauser-Horne inequality:

$$\text{(CH)}\ -1 \leq P(A_1B_1) + P(A_2B_1) + P(A_2B_2) - P(A_1B_2) - P(A_2) - P(B_1) \leq 0$$

Given the uniform angular separations in the case[26] we are considering, we have, according to quantum mechanics:

$$\text{(QM-1)}\ P(A_1B_1) = P(A_2B_1) = P(A_2B_2) = Q = 1/2 \cos^2\theta,$$

$$\text{(QM-2)}\ P(A_1B_2) = Q' = 1/2 \cos^2 3\theta$$

By the symmetry of the singlet state we have:

$$\text{(QM-3)}\ P(A_1) = P(A_2) = P(B_1) = P(B_2) = 1/2$$

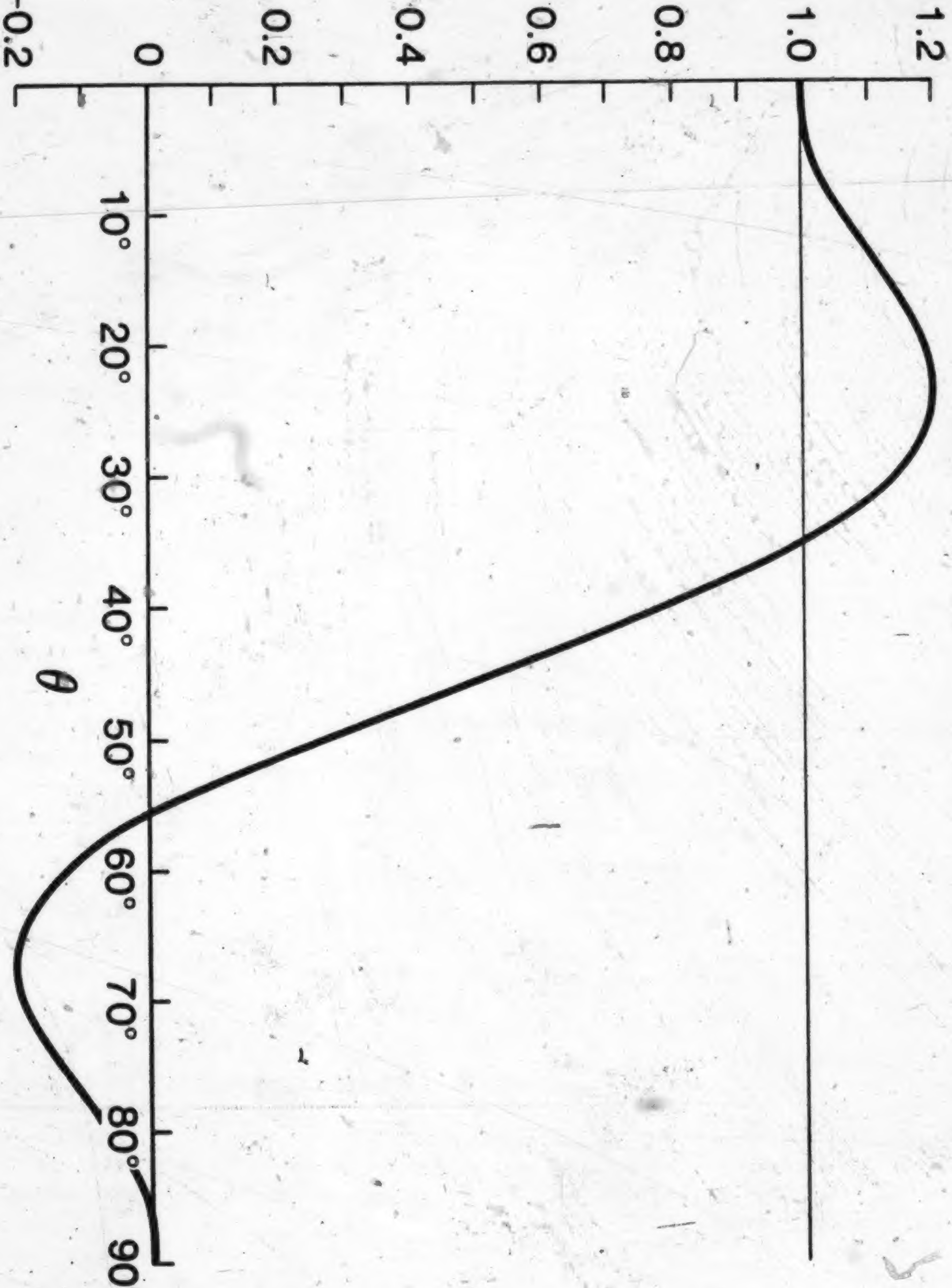
By substituting the probabilities Q and Q' into the inequality (CH) we get the simplified Clauser-Horne inequality (SCH):

$$\text{(SCH)}\ 0 \leq 3Q - Q' \leq 1$$

In Figure 3-2, 3Q - Q' has been plotted for values of θ varying from 0° to 90°. The inequality (SCH) is violated in the region $\theta \approx 0°$ to $\theta \approx 34°$ and in the region $\theta \approx 56°$ to $\theta \approx 90°$. The maximum violations are at $\theta = 22.5°$ and $\theta = 67.5°$.

Since (SCH) is a necessary condition for there to be a Bell local hidden variables theory for the experiment under analysis, and since quantum mechanical probabilities violate (SCH), it would appear that Bell local hidden variables theories of the experiment under analysis are impossible. Inasmuch as experiments confirm the predictions of quantum mechanics over theories constrained by (SCH), this conclusion may be said to have gained some empirical support.[27]

FIGURE 3-2 Graph of 3Q - Q'.



SECTION FIVE: THE EVIDENCE

At the time of writing, the experimental testing of quantum mechanical predictions for spin correlation statistics has proven to be extremely difficult.[28] Conditions for a direct test of (SCH) have not yet been met. Furthermore, in experiments which have been performed, various inefficiencies typically reduce the curve of Figure 3-2 to a curve well within the limits set by (SCH); and it is only by extrapolation of this curve to allow for inefficiencies (which itself is no simple matter) that the curve of Figure 3-2 is deemed to receive any confirmation by extant data. In view of this, one suspects that Shimony is overstating things when he notes of Bell's work:

> Since the experiments inspired by his work overwhelmingly support quantum mechanics, it follows that no contextual hidden variables theory is viable unless it postulates a kind of non-locality.[29]

Similarly, van Fraassen comments of Bell's inequality:

> Quantum mechanics predicts the violation of this inequality. And finally, experimentation so far has produced overwhelming support for the quantum mechanical predictions.[30]

Once again, this seems to be an extremely generous interpretation of just what it is that the data establishes.

In support of this contention that there has been a widespread tendency to overstate the evidential support for Bell's theorem, Marshall et. al.[31] show that the random variable/phase space apparatus can be used to produce local models according well with the extant data--data derived in experiments which have counters of the order of 15-20% efficiency. Their model shows that there is a great deal of wishful thinking used in claims that experiments support the predictions of quantum mechanics and violate the predictions of Bell local hidden variables theories by up to 40 standard deviations. The model produced by Marshall et al. collapses for high efficiency experiments. Alas, extant experiments are not high efficiency experiments.

In the opinion of Clauser and Shimony[32] the most reliable tests of Bell's inequality are low energy photon cascade experiments where photon polarization rather than spin is measured for photon-pairs in appropriate singlet states. These authors note that a direct test of spin has been performed on proton pairs. Clauser and Shimony perceive this test to be among the least reliable of the experiments.

A recent proposal of considerable interest is that of Aspect, Dalibard and Roger.[33] In his seminal paper, Bell notes in his concluding comments the importance of changing the analyzer settings while the particles are in flight--or else locality could be maintained on the ground that preset analyzer settings could influence the state of affairs at the source without any recourse to superluminary effects, (in essence, system-apparatus independence might be violated, so that the requirement of Bell locality would not be justified). Aspect et. al. have performed tests in which the analyzer settings were changed while the particles are in flight. They found results which appear to accord well with quantum mechanics. It must be noted, however, that extrapolations are still involved. Furthermore, Marshall et al. have raised concerns about the data analysis of the Aspect experiment.

The predictions of quantum mechanics may thus be said to have received ambiguous support from experiments so far performed. The ambiguity is due primarily to experimental inefficiencies. While this situation may change in the next generation of quantum correlation experiments, claims that Bell's Theorem has been unambiguously well-confirmed are premature.

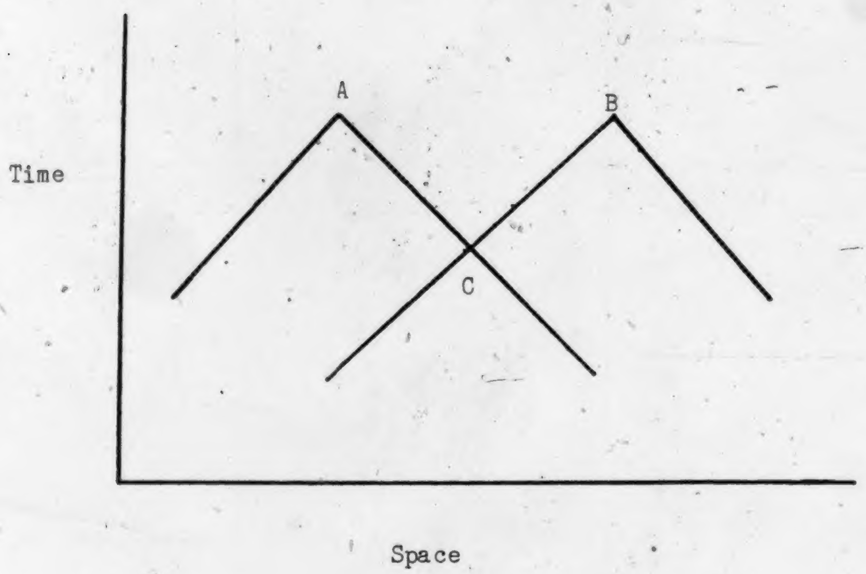
SECTION SIX: DETERMINISM AND STOCHASTICITY

It has become commonplace to view the Bell result as having important implications for the debate between determinists and indeterminists.[34] Some of these implications will now be presented and assessed.

Van Fraasen,[35] for instance, wishes to apply the Bell result to the 'common cause' model of explanation. Briefly put (see Figure 3-3), if events A and B are correlated (e.g., measurement results in a Bell-type experiment) and are also spacelike separated, then a common cause for the correlation will be some factor C in the past (in the intersection of the backward light cones of A and B). I will have more to say about the 'common cause' model of explanation in Chapter Five; what matters here is that consideration of common causes, (lying in the intersection of the backward light cones of spacelike separated but correlated events A and B), have led some theorists[36] to think of particle-pairs as being emitted from the source in microstates $\lambda \in \Lambda$.

Deterministic hidden variables theorists say that the $\lambda \in \Lambda$ at the source 'determine' measurement outcomes at the spatially distant analyzers. On the assumption of Bell locality we know from Bell's theorem that there can be no phase space probability density $\rho(\lambda)$ which will yield the quantum spin correlation measurement statistics. On

FIGURE 3-3 Backward Light Cones for Events A and B.



(Note: the event C lies in the intersection of the backward light cones of events A and B.)

the assumption of Bell locality, there will be no $\lambda \in \Lambda$ at the source which determine measurement outcomes at the analyzers. Thus Hellman has commented:

> It is by now widely recognized that a powerful case for the indeterministic or ultimately random behaviour of certain quantum mechanical systems can be based on Bell's Theorem.[37]

In a similar vein, Suppes, who wishes to support the claim that the fundamental laws of natural phenomena are probabilistic in character, notes:

> First, within the framework of classical physics there is evidence in favour of randomness in radioactive decay. Second there is the extensive literature on the possibility of hidden variables theories as a theoretical underpinning of quantum mechanics, — theories formulated to restore determinism as conceived within the framework of classical physics. I want to examine each of these lines of attack because it seems to me that the outcome of the analysis of either makes it extremely difficult to believe that the universe operates in a deterministic fashion.[38]

Hellman and Suppes are correct. Treating Bell's theorem as a 'no hidden variables' proof does make the Bell result relevant to the determinist position.

In Chapter One it was seen that Laplacean determinism had two components: (a) deterministic equations of motion, and (b) determinate and complete states. Treating the Bell result as a 'no hidden variables' result means that on the assumption of Bell locality there can be no classically determinate states $\lambda \in \Lambda$. Bell's theorem, on this view, rules out something necessary for a Laplacean deterministic account of the quantum spin correlation measurement statistics: determinate and complete states λ.

Suppes, of course, also cites radioactive decay as providing grounds, even within classical physics, for believing there is indeterminism in nature. The case of radioactive decay is worth contrasting with the Bell case. Both cases appear to undermine Laplacean determinism, but they do so in different ways.

Consider a classical treatment of radioactive decay.[39] The systems under analysis are classical particle-systems (and so are classically determinate). The question is whether the radioactive decay process is deterministic or indeterministic.

Suppose at time t there are N atoms of a given radioelement. The decay equation is:

$$1) \quad -\frac{dN}{dt} = cN,$$

where c is a constant called the radioactive constant. The constant is a property of a given radioelement (so its value changes from species to species). Since dN is the number of atoms disintegrating in unit time dt, dN/N will represent the fraction of the total number of atoms which decay in this time. Dividing by dt and including the minus sign gives:

$$2) \quad -(dN/N)/dt,$$

which will be the fraction of atoms present decaying in unit time. From (1) above, it is clear that this quantity is equal to c, so the decay constant is actually the fraction of the total number of atoms of a given radioelement decaying in unit time.

The fundamental law of radioactive decay--enunciated by Rutherford and Soddy in 1903 (fourteen years before any firm connection between radioactive decay and the quantum theory was established)--states that the decay constant c is independent of the physical condition (including temperature and pressure) and state of chemical combination of its corresponding radioelement. The rate of radioactive decay of a given species depends on the decay constant c associated with that species and upon nothing else. Apparently it is not possible to causally influence the rate of decay.

This suggests that there is a definite probability, determined by c, that an atom will disintegrate at a particular time. The life of any atom may be anything from 0 to ∞. According to this classical account of radioactive decay it is impossible to tell in advance whether or not a particular atom will disintegrate. What is known--and all that can be known, according to the fundamental law of radioactive decay--is that a definite fraction c of atoms will decay in unit time. The decay process is indeterministic. Since the decay constant c is independent of the physical and chemical state of its corresponding radioelement, the indeterminism does not arise because of our ignorance of significant physical or chemical factors relating to radioelements. Equation (1) above thus defines a stochastic process.

There are thus two ways in which Laplacean determinism can fail: (1) the system states may not be determinate and complete; (2) the equations of motion may not be deterministic. Given the assumption of Bell locality, it is a consequence of Bell's theorem that we cannot have determinate and complete states $\lambda \in \Lambda$.

Now if Bell local Laplace-deterministic theories are ruled out by the Bell result, could there nevertheless be Bell local stochastic hidden variables theories? The answer, I will argue, is no. In a stochastic hidden variables theory the states $\lambda \in \Lambda$ at the source only determine probabilities for outcomes at the analyzers (where these are not 0 or 1 probabilities familiar from deterministic hidden variables theories). Of such theories, Hellman has commented:

> . . . it is not even clear how to formulate 'physical locality' for stochastic theories in a way that allows one to decide the connection between it and the mathematical conditions needed to derive a Bell-type inequality violated by quantum predictions.[40]

I think Hellman is correct. However, there is a more important point to be made. If such stochastic hidden variables theories are committed to the existence of states $\lambda \in \Lambda$ which determine, at the time of measurement, values for all spin components,[41] then such theories cannot be Bell local.

The picture is this: particle-pairs are emitted from the source in classically determinate states λ. These states develop over time in accord with a stochastic equation of motion. The state of the pair-system is classically determinate at all times, but the state λ_t at the time t of emission does not determine the state λ_{t+1} at the time of measurement. What matters for the Bell argument is classical determinacy of state. The stochastic hidden variables strategy makes the assumption of classical determinacy. But any theory, be it stochastic or deterministic, which has a commitment to classically determinate states $\lambda \in \Lambda$ cannot be Bell local.

FOOTNOTES

[1] Bell (1964), p. 194.

[2] Einstein, Podolsky and Rosen (1935). See also Furry (1936).

[3] These are versions of the original assumptions and not the original assumptions themselves. I believe, however, that my way of stating the assumptions removes ambiguities from the original statement of the assumptions.

[4] Bohm (1951), Ch. 12. See also Bohm and Aharonov (1957).

[5] Bell (1964).

[6] After all, we could have measured any spin component or particle.

[7] Bohr (1958), p. 311.

[8] Bell (1964).

[9] $S_1(\vec{a})$ = spin along a on particle 1, etc.

[10] Again, as in the EPR argument, we are considering pairs of particles emitted from a common source and prepared in the singlet spin state ψ_0. Each pair of particles separates freely until they reach their respective Stern-Gerlach magnets, which are used to measure selected components of spin.

[11] $A(\vec{a},\lambda)$ is the result of measuring the spin of particle 1 along $\vec{a}$. $B(\vec{b},\lambda)$ is the result of measuring the spin of particle 2 along $\vec{b}$.

[12] See note (10).

[13] See Bell (1964).

[14] Fine (1981) makes the distinction between Bell locality (which he refers to as a factorizability condition) and physical locality (i.e., Einstein locality). That there is a distinction here has been very important to the development of my ideas on this topic.

[15] Bell (1964), p. 199.

[16] Shimony (1984), p. 25. Shimony seems to suppose that violations of Bell locality are always violations of Einstein locality.

[17] Bell's original argument (1964) establishes that no theory that is classically determinate as regards spin component measurement results can be Bell local.

[18] Sharp has suggested in conversation that this is the case provided there is some 'no disturbance' principle in operation as regards measurements of spin components. The following considerations make it appear natural to assume some 'no disturbance' principle: assume there was disturbance when measurement took place so that the measured value for spin along $\overleftarrow{a}$ was not necessarily the value for spin along $\overleftarrow{a}$ immediately prior to or independently of measurement. Since $S_1(\overleftarrow{a}) = +1$ iff $S_2(\overleftarrow{a}) = -1$, and 1 and 2 are spacelike separated, the introduction of such disturbances seems to violate intuitions about physical locality--for the disturbance of 1 to yield the measured value +1 must constitute a disturbance of spacelike separated 2 to get a measured value -1.

[19] Bell (1964), p. 195.

[20] Bohm and Hiley (1982).

[21] Ibid., pp. 343-4.

[22] Clauser and Horne (1974).

[23] Fine (1981).

[24] Sharp and Shanks (1985).

[25] Note that the directions $\overleftarrow{a}_i$, $\overleftarrow{b}_i$, etc. are in the plane transverse to the direction of flight of the particles. In this special case the angles between $\overleftarrow{a}_1\overleftarrow{b}_1$, $\overleftarrow{b}_1\overleftarrow{a}_2$, and $\overleftarrow{a}_2\overleftarrow{b}_2$ are all θ and the angle between $\overleftarrow{a}_1\overleftarrow{b}_2$ is 3θ for various values of θ.

[26] See note (25).

[27] However, see the next section for a brief analysis of the evidence in support of Bell's theorem.

[28] Some of the problems which face the experiments are discussed in Clauser and Shimony (1978). See also Marshall et. al. (1983).

[29] Shimony (1984), p. 26.

[30] van Fraassen (1982), p. 35.

[31] Marshall et. al. (1983).

[32] Clauser and Shimony (1978).

[33] Aspect et. al. (1982).

[34] Fine (1981); van Fraasen (1982), Hellman (1982a); Suppes (1984).

[35] van Fraasen (1982).

[36] van Fraasen (1982), Fine (1981).

[37] Hellman (1982a), p. 445.

[38] Suppes (1984), p. 221.

[39] Glasstone (1967), pp. 143-145.

[40] Hellman (1982b), p. 463.

[41] We have seen that it is enough to consider four spin components $\vec{a}_1$, $\vec{a}_2$, $\vec{b}_1$ and $\vec{b}_2$.

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CHAPTER FOUR

ARTHUR FINE ON BELL'S THEOREM

> I attempt to throw cold water on Fine's hopes for a classical understanding of quantum theory by focussing on his treatment of the problem of joint distributions.
>
> A. Stairs[1]

SECTION ONE: INTRODUCTION

In this Chapter the question whether Bell's Theorem is an inevitable consequence of the use of classical probability theory in the analysis of quantum spin correlation measurement statistics, is studied with specific reference to the work of Arthur Fine. While the conclusions of the analysis will be largely confined to Fine's work, it must be borne in mind that that work has, over the last decade, been very influential among theorists working on the interpretation of quantum mechanics.

In quantum mechanics joint probability distributions are not generally well-defined for non-commuting observables.[2] A recurrent theme in Fine's work is that while all quantum mechanical observables must take simultaneous exact values, those values are to be distributed just as they are in quantum mechanics--for example, if A and B are non-commuting observables, they will take simultaneous exact values but there will be no joint probability distribution P^{ϕ}_{AB}. The probabilistic constraint on much of Fine's work on hidden variables interpretations of quantum mechanics is the avoidance of commitment to joint probability distributions where these joint distributions are not quantum mechanically well-defined. This point cannot be emphasized enough, for in his work[3] he has repeatedly claimed that what underlies the Bell result is not the hidden variables assumption, or the Einstein locality assumption. Rather, what he views as causing the trouble is the idea that in a hidden variables theory of the Bell-type correlation statistics, joint distributions must be interpolated for non-commuting observables. Clearly, if we use the random variable/phase space apparatus and represent non-commuting observables by random variables on a common phase space then we are trivially committed to the existence of joint distributions for them. Fine believes, then, that the force of the Bell argument may be subverted by construing the Bell-type probabilities 'classically' and yet in such a way as to avoid the interpolation of joint distributions for non-commuting observables. His Statistical Random Variable Model and Prism Model approaches alter the usual probabilistic setting to provide "classical" but "local" accounts of the Bell statistics. The Synchronization Model approach retains the usual random variable/phase space apparatus but employs additional variables--analyzer orientation dependent retardations--to account for the correlation statistics.

SECTION TWO: THE STATISTICAL RANDOM VARIABLE APPROACH

The statistical random variable (SRV) approach begins with an explicit rejection of the random variable/phase space apparatus.[4] On the constructive side of things, the SRV approach proceeds as follows:

1) With each quantum state $\emptyset$ associate a set $V^{\emptyset}$ of functions which assign each quantum observable A one of its eigenvalues. Where

$x \in V^{\emptyset}$, let $x(A)$ be the value x assigns to A. All quantum observables will thus take simultaneous exact values for each x. The functions x thus play the role of classically determinate states in this theory.

It remains for Fine to reconcile the values we have interpolated for all quantum observables and each quantum state $\emptyset$ with the probabilities defined by quantum mechanics. He proceeds as follows:

2) Associate each observable A with a classical probability space by letting:

a) $V_A^{\emptyset} = [x(A) : x \in V]^{\emptyset}$

b) $B_A^{\emptyset}$ = the Borel subsets of $V_A^{\emptyset}$

c) $P_A^{\emptyset}$ = the probability measure which the quantum state $\emptyset$ associates with quantum observable A according to quantum mechanics. (That is, $P_A^{\emptyset} = (\emptyset, A\emptyset)$).

We then have:

d) Each observable A is to be associated with its own (private) classical probability space:

$S_A^{\emptyset} = < V_A^{\emptyset}, B_A^{\emptyset}, P_A^{\emptyset} >$

In this theory, each observable A takes a value for any classically determinate state x associated with a quantum state $\emptyset$, the values of the observables being distributed among the classically determinate states according to the rules of quantum theory.[5]

So far, nothing has been said about joint distributions. Since the observables in an SRV approach are not associated with random variables on a common phase space, there are, at this stage of the game, no joint distributions defined at all. Fine comments:

> For pairs (triples or whatever) of statistical variables there is no way to derive joint distributions. Say for quantities A, B a joint distribution would be a nice measure on the Borel subsets of $V^{\emptyset}_A \times V^{\emptyset}_B$ that satisfies the marginal probability conditions so as to give us $P^{\emptyset}_A$ and $P^{\emptyset}_B$ back again. Although mathematically there always exists such nice measures, nothing assigned to the values in $V^{\emptyset}_A$ and $V^{\emptyset}_B$ bear on their joint occurrence in a way that would single out one of them as the joint distribution. Of course, nothing so far rules out the existence of joint distributions either. So far the question is left entirely open.[6]

Fine suggests that we should once again take the lead of quantum mechanics when it comes to the question of the existence of joint distributions. Joint distributions, where they exist, are to coincide with quantum mechanically specified distributions of observables satisfying the following rule:[7]

(jd) Observables $A_1, \ldots, A_n$ of a quantum system have a joint distribution just in case, corresponding to every n-place Borel function f, there is an observable of the system with operator $f(\mathbf{A}_1, \ldots, \mathbf{A}_n)$, and corresponding to every state $\emptyset$ of the system there is a measure $P^{\emptyset}_{A_1, \ldots, A_n}$ on the Borel subsets of R^n that returns the quantum single distributions $P^{\emptyset}_{A_i}$ as marginals, such that:

$$P^{\emptyset}_{A_1, \ldots, A_n}(f^{-1}(S)) = P^{\emptyset}_{f(A_1, \ldots, A_n)}(S)$$

for every Borel set S of reals and quantum state $\emptyset$.

Fine[8] offers a proof that (jd) can be satisfied only if the observables $A_1, \ldots, A_n$ commute in pairs. Fine says that A and B have a joint distribution construed as a quantum mechanically well-defined measure on the Borel subsets of $V_A^0 \times V_B^0$ just in case A and and B commute. He comments:

> If we require (jd) . . . then the framework of simultaneous value assignments yields the joint distributions of quantum mechanics (and no others). Thus this framework fulfills all the requirements of the completeness programme.[9]

And, significantly, he also notes:

> Perhaps, then, we ought to accept the straight-line induction; that where (jd) fails, and quantum mechanics does not give a well-defined joint probability distribution, neither would experiments. After all, if we hold that probabilities (including joint probabilities) are real properties, then some observables may simply not have them.[10]

In the SRV approach we get simultaneous exact values for all observables. What we do not get is the commitment to joint probability distributions for all these observables. How does this help in analyzing the Bell case?

Recall the Bell-type experiment discussed in Chapter Three. It was seen there that the Bell-type inequality (SCH), violated by quantum predictions, was a constraint upon phase space probabilities to the effect that if A_1, A_2, B_1 and B_2 were treated as random variables on a common phase space then there would have to be some distribution $P_{A_1A_2B_1B_2}$ which yields the various marginal distributions.

According to Fine, the violation of the Bell-type inequality (SCH) shows merely that (jd) fails for non-commuting observables. The violation of (SCH) shows that we must not interpolate joint distributions for A_1A_2-pairs and for B_1B_2-pairs. As Fine notes:

> Of course, it is Bell's important and lasting contribution to have found cases especially simple, and also experimentally tractable, where (jd) does fail.[11]

In the SRV approach we have simultaneous exact values for all observables (i.e., hidden variables or classical determinacy). Bell locality cannot be formulated. Since the observables are not treated as random variables on a common phase space, there is no longer a requirement to retrieve the quantum spin correlation measurement statistics from a common density $\rho(\lambda)$.[12] Notwithstanding this, it is hoped that the theory is physically local or Einstein local.

In the SRV approach to the Bell issue we consider the experimental observables as being associated with the following cl~~assical~~ probability spaces:

1) $A_i \cdot S^{\emptyset}_{A_i} = \langle V^{\emptyset}_{A_i}, B^{\emptyset}_{A_i}, P^{\emptyset}_{A_i} \rangle$

$B_j \quad S^{\emptyset}_{B_j} = \langle V^{\emptyset}_{B_j}, B^{\emptyset}_{B_j}, P^{\emptyset}_{B_j} \rangle$

Thus, in the singlet state $\emptyset$ all observables on a pair-system will take simultaneous exact values and these values will be distributed just as they are in quantum mechanics. Quantum mechanics only defines joint distributions for the commuting A_iB_j-pairs. A joint distribution in a SRV approach will be a probability measure, in each case, defined as the measurable (i.e. Borel) subsets of $V^{\emptyset}_{A_i} \times V^{\emptyset}_{B_j}$. According to the rule (jd), there is no joint distribution which is quantum mechanically well-defined on the measurable subsets of of $V^{\emptyset}_{A_i} \times V^{\emptyset}_{A_k}$ for non-commuting A_i, A_k. According to Fine, evidence which supports a violation of Bell's inequality is evidence that (jd) cannot be satisfied for all of the observables constrained by (SCH), and thus is evidence that certain observables, the A_iA_k and

B_jB_1-pairs, cannot have a well-defined joint distribution. Is this SRV approach paradise regained? It will be argued below that it is not.

First, and to be fair to Fine, it must be noted that recently he has been critical of the SRV approach to Bell-type problems:

> . . . since quantum theory mandates different probability measures for different experimental arrangements . . . we should not suppose that the same distribution $\rho(\lambda)$ applies to A_1B_1 *and* to A_2B_1 *and* to A_2B_2 *and* to A_1B_2--for no two of these are compatible. Yet short of the sort of communication between particles at the barriers that would violate physical locality, it is difficult to see how the distribution of properties at the source could be altered to correspond to these distinct joint experiments.[13]

This worry of Fine's, however, relies on the assumption of system-apparatus independence.[14] If it were the case that systems were emitted from the source in certain classically determinate states λ if and only if certain experiments were going to be performed on them, then the distribution ρ *could* be assumed to vary in a way that would correspond to distinct joint experiments but not involve violations of Einstein locality. That is, Fine's worry above is undermined if the principle of system-apparatus independence fails.

Sadly, however, there is a more fundamental objection to the SRV approach to the Bell problem. What is to be made of the claim that non-commuting observables A_i and A_k do not have a well-defined joint distribution? From an operational point of view one will have single frequencies associated with $P^0_{A_i}$ and $P^0_{A_k}$. To use Cartwright's image,[15] the failure of (jd) will show up something like this: we may chart finite histograms for the joint values of A_i and A_k, but those values will "forever bounce about" and will not converge to a single curve. Thus Cartwright comments:

> There is by hypothesis some joint frequency in every finite collection, but these frequencies do not approach any limit as the collections grow large. This may not seem surprising in a completely chaotic universe, but it is very surprising here where the marginal probabilities are perfectly well-defined.[16]

Now given Fine's claim that all observables take simultaneous exact values at any time, we may anticipate that there will be a joint frequency corresponding to $P_{A_1A_2B_1B_2}$ in every finite collection--even if these frequencies do not approach any limit as collections grow large. But this is precisely what the Bell result rules out--otherwise there would be no point in performing Bell-type experiments which concern finite collections and which aim at demonstrating violations of Bell-type inequalities on the basis of the statistical properties of these finite collections. Were the Bell-type experiments to provide unambiguous evidence of violations of Bell-type inequalities they would rule out also the SRV approach.

Perhaps another way to put this is as follows: in order to make sense of Fine's denial of the existence of joint distributions for non-commuting A_i, A_k, we must take it that he rules out joint distributions only in the limit of collections tending to ∞ (by Cartwright's argument), while being committed, because of the alleged classical determinacy, to the existence of joint distributions--i.e., joint frequencies--in finite cases (once again, by Cartwright's argument). But it is the existence of joint frequencies for non-commuting observables (e.g., A_1 and A_2; B_1 and B_2) which the Bell theorem rules out.

there is no distribution $\rho(\lambda)$ which will return even the finite statistics for the various A_iB_j-experiments.

In orthodox quantum mechanics, where it is standard to deny that non-commuting observables have simultaneous exact values, the claim that such observables do not have a well-defined joint distribution will come as no great surprise--the joint occurrence of values is ruled out as a matter of course. This feature concerning probability distributions is much more surprising in a theory which claims that all observables on a system do take simultaneous exact values. In this case, speculation about the probabilities for the joint occurrence of values makes good sense. Fine's picture of ontological completeness (i.e., classical determinacy) is hard to reconcile with his views concerning the failure of (jd).

Consider an ensemble of systems each with a value a_i for some observable A. Given Fine's ontological picture of classically determinate systems, observables B and C (non-commuting with A) will take precise values for each system in the ensemble. But this picture, from a conceptual point of view, carries with it a commitment to joint distributions: we can ask for the probability that $\mathrm{Val}(A)_\emptyset = a_i$ and $\mathrm{Val}(B)_\emptyset = b_j$, for instance. The question Fine never really answers is why we take some joint occurrences seriously and why we ignore others: even if quantum mechanics does not define joint distributions for non-commuting observables, surely the ontological picture Fine employs requires them.

SECTION THREE: THE PRISM MODEL APPROACH

Fine's prism model approach to the analysis of quantum spin correlation measurement statistics is a further attempt to provide a 'classical' analysis of quantum spin correlation measurement statistics which has only those joint distributions as defined by orthodox quantum mechanics. Fine's prism model approach is discussed at several points in his work.[17] In a recent publication,[18] Fine has attempted to interpret some of Einstein's views about quantum mechanics in terms of the prism model concept. The prism model approach has been subject to lengthy and detailed criticism in a recent work by Sharp and Shanks.[19] I will review here the concept of a prism model and the main points of criticism.

Fine's aim in producing a prism model analysis of quantum spin correlation measurement statistics is to preserve the concepts of determinism and physical locality in the face of the Bell result. The essential feature of a prism model analysis is the use of partial random variables on the phase space Λ to represent observables on a quantum system.

Referring back to the experimental version of Bell's argument in Chapter Three, then with each of the four response functions A_i, B_i ($i = 1,2$) Fine associates a proper subset $\sigma(A_i)$, $\sigma(B_i)$ of which is said to be the category of the corresponding response function.[20] Fine's idea is that the category of a response function consists of those classically determinate states $\lambda \in \Lambda$ such that a particle in that classically determinate state is suitable for a measurement of the corresponding response function. As Fine puts it:[21]

1) $\lambda \in \sigma(A_i)$ iff there is some probability (perhaps 0) that an A-particle (the one in the A-wing) in state λ will pass an A-analyzer set in its i^{th} position and then be counted.

Fine regards it as a category mistake (comparable to asking what the probability is that a liquid is harder than a gas) to speak of any probability (even 0) for an A-particle in state λ to pass the A-analyzer set in its i^{th} position, when $\lambda \notin \sigma(A_i)$.

Thus, in a prism model the response functions are treated as partial random variables, e.g., $A_i : \sigma(A_i) \longrightarrow [0,1]$, partial since A_i is only defined for a proper subset $\sigma(A_i)$ of Λ. Fine defines A_i as follows:[22]

2) $A_i(\lambda) = 1$ iff an A-particle in state λ will produce a count in the A-wing when the A-analyzer is in its i^{th} position.

$A_i(\lambda) = 0$ iff $A_i(\lambda) \neq 1$ (and $\lambda \in \sigma(A_i)$).

Fine's prism model approach is deterministic since he regards particle-pairs as being emitted from the source in classically determinate states λ. These λ determine at the source the responses of particles to orientations of the spacelike separated analyzers.

The probability for a result in a prism model is not the probability that a system in the singlet state per se will produce that result. It is rather that a system in an appropriate category of classically determinate states will produce that result. The probability $P(A_i)$, which is the probability for a count in the A-wing when the A-analyzer is in its i^{th} position, is no longer calculated as

$\int \rho(\lambda)d\lambda$ over the region of Λ where $A_i(\lambda) = 1$. Rather, this probability appears in a prism model as a conditional probability $P(A_i(\lambda) = 1 | \lambda \in \sigma(A_i))$, i.e., as a ratio of integrals:

$$3)\quad P(A_i) = \frac{\int_{\Lambda_{A_i}} \rho(\lambda)d\lambda}{\int_{\sigma(A_i)} \rho(\lambda)d\lambda}$$

where $\Lambda_{A_i} = [\lambda : A_i(\lambda) = 1]$.

Similarly $P(A_iB_j)$ is treated as a conditional probability:

$$4)\quad P(A_iB_j) = P(A_i(\lambda) = B_j(\lambda) = 1\ \lambda \in \sigma(A_i) \cap \sigma(B_j)),$$

and hence as:

$$5)\quad P(A_iB_j) = \frac{\int_{\Lambda_{A_iB_j}} \rho(\lambda)d\lambda}{\int_{\sigma(A_i)\cap\sigma(B_j)} \rho(\lambda)d\lambda}$$

where $\Lambda_{A_iB_j} = [\lambda : A_i(\lambda) = B_j(\lambda) = 1]$.

In a prism model joint distributions may be undefined--for instance, when $\sigma(A_i) \cap \sigma(B_j) = \emptyset$, $\emptyset$ the empty set.

Fine has produced two sorts of prism model: minimal models and maximal models. The root idea behind Fine's minimal models[23] is that categories for incompatible observables (non-commuting observables) be disjoint. Thus:

$$6)\quad \sigma(A_1) \cap \sigma(A_2) = \emptyset = \sigma(B_1) \cap \sigma(B_2)$$

Fine chooses $\Lambda = [0,4]$ with λ uniformly distributed over this closed interval.[24] Categories are assigned as follows:

7) $\sigma(A_1) = [0,2]$, $\sigma(A_2) = (2,4)$

$\sigma(B_1) = [0,1] \cup (2,3)$, $\sigma(B_2) = [1,2] \cup (3,4]$

Fine defines step functions $f(x)$, $g\theta(x)$ and $g\theta'(x)$ on $[0,1]$ as follows:

8) $$f(x) = \begin{cases} 1, & 0 \leq x \leq 1/2 \\ 0, & 1/2 < x \leq 1 \end{cases}$$

9) $$g\theta(x) = \begin{cases} 1, & 0, \leq x \leq Q \\ 0, & Q < x \leq 1/2 \\ 1, & 1/2 < x \leq 1-Q \\ 0, & 1-Q < x \leq 1 \end{cases}$$

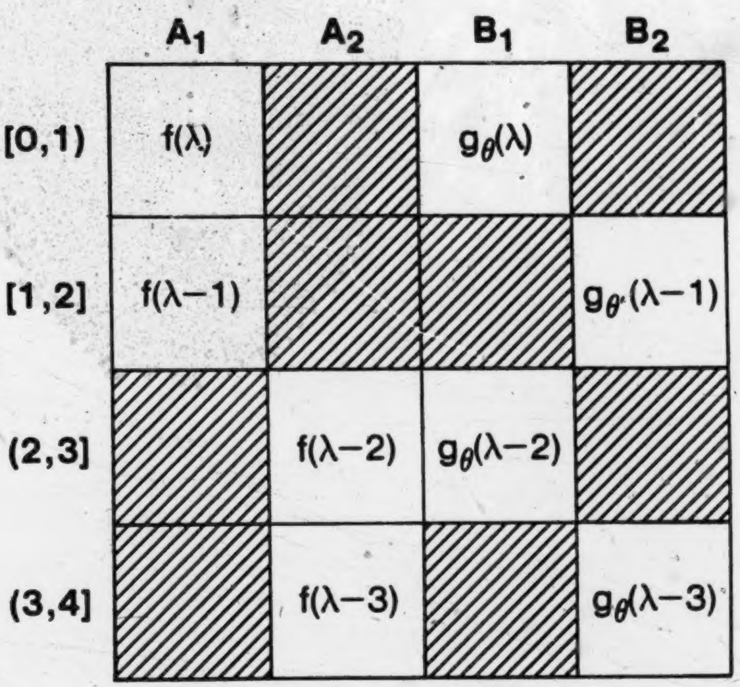
(the function $g\theta'(x)$ is defined like $g\theta$ except that Q' replaces Q. As in Chapter Three, $Q = 1/2 \cos^2\theta$, $Q' = 1/2 \cos^2 3\theta$). The response functions are then defined in terms of these step functions. The definitions are given in Figure 4-1.

Reproducing Fine's examples of probability calculations in a minimal model, we have:

10) $$P(A_1) = \frac{1/4 \left[\int_0^1 f(\lambda)d\lambda + \int_1^2 f(\lambda-1)d\lambda \right]}{1/4 \int_0^2 d\lambda}$$

$$= \frac{1/2 + 1/2}{2} = 1/2$$

FIGURE 4-1 Definition of Response Functions in the Minimal Model.



	A_1	A_2	B_1	B_2
[0,1)	$f(\lambda)$	▨	$g_\theta(\lambda)$	▨
[1,2]	$f(\lambda-1)$	▨	▨	$g_\theta(\lambda-1)$
(2,3]	▨	$f(\lambda-2)$	$g_\theta(\lambda-2)$	▨
(3,4]	▨	$f(\lambda-3)$	▨	$g_\theta(\lambda-3)$

11) $$P(A_1B_2) = \frac{1/4 \int_1^2 f(\lambda-1)g\theta'(\lambda-1)d\lambda}{1/4 \int_1^2 d\lambda} = Q'$$

Fine describes his maximal models as being the best one can do, within the prism model framework, without letting in enough apparatus to derive Bell-type inequalities violated by quantum spin correlation measurement statistics.[25] In these models, three out of four single response functions may be defined for measurable subsets of λ--but not all four at once.

In these models $\Lambda = [0,4]$ and ρ is the same uniform density function. The categories for the response functions are given in Figure 4-2. Once again, probabilities are calculated as conditional probabilities. Reproducing Fine's examples:

12) $$P(B_1) = \frac{1/4 \left[\int_0^1 g\theta(\lambda)d\lambda + \int_1^2 g\theta(\lambda-1)d\lambda + \int_2^3 f(\lambda-2)d\lambda\right]}{1/4 \int_0^3 d\lambda}$$

$$= \frac{1/2 + 1/2 + 1/2}{3} = 1/2$$

13) $$P(A_1B_2) = \frac{1/4 \left[\int_0^1 f(\lambda)g\theta'(\lambda)d\lambda + \int_3^4 f(\lambda-3)g\theta'(\lambda-3)d\lambda\right]}{1/4 \left[\int_0^1 d\lambda + \int_3^4 d\lambda\right]}$$

$$= \frac{Q' + Q'}{1 + 1} = Q'$$

FIGURE 4-2 Definition of Response Functions in the Maximal Model.

	A_1	A_2	B_1	B_2
[0,1)	$f(\lambda)$	▨	$g_\theta(\lambda)$	$g_{\theta'}(\lambda)$
[1,2]	▨	$f(\lambda-1)$	$g_\theta(\lambda-1)$	$g_\theta(\lambda-1)$
(2,3]	$g_\theta(\lambda-2)$	$g_\theta(\lambda-2)$	$f(\lambda-2)$	▨
(3,4]	$g_{\theta'}(\lambda-3)$	$g_\theta(\lambda-3)$	▨	$f(\lambda-3)$

As can be seen from Figure 4-2, for the maximal models, unlike the minimal models, joint probabilities are produced for non-commuting observables such as A_1 and A_2, as well as for the commuting observables, such as A_1 and B_1.

Fine[26] initially regarded his prism models not as general accounts of spin correlation measurement statistics but as being models for the restricted types of experiments discussed in Chapter Three.[27] Lately he has been much less cautious, regarding prism models as a precise formalization of some of Einstein's views on the quantum theory. For instance he has recently commented:

> It seems to me, therefore, that we have excellent reasons to challenge the received opinion that Bell's Theorem rules out Einstein's statistical interpretation. For the interpretation that it rules out, the idea of an ensemble representation, can only be attributed to Einstein on the basis of a contrived interpretive dance. There is, moreover, another interpretation, the idea of a prism model, that does seem to fit Einstein's various remarks snugly, and that is not subject to Bell's Theorem.[28]

The criticisms of the prism model programme to be discussed below are justified even on the cautious reading of the prism model enterprise, since in assessing the adequacy of a model for any experiment, it is surely legitimate to ask how the model fares if a straightforward extrapolation of the scope of the experiment is made, or if minor modifications are effected. In what follows, both minimal and maximal models will be criticized in terms of their implications for particle-rejection rates in correlation experiments.

As has been seen, the basic idea behind a minimal model is disjoint categories for non-commuting observables. For experiments which involve two settings for each analyzer, the requisite disjointness of categories could be achieved at the expense of a 50% single rejection rate--50% of the emitted particles must be defective for any given analyzer orientation. In virtue of being defective, they will be in the wrong category for the experiment being run and will simply not show up. Rejection rates in A_iB_j-experiments will be 75%.[29]

Joint detection experiments performed to date have been (in terms of particles detected) of the order of 20% efficient at best. Prism models in their minimal formulation accord well with the extant evidence--though they do not match well with the usual account of the inefficiency in terms of mechanical (as opposed to prism-theoretic) inefficiency.

Experiments performed to date have typically involved only four directions. There is no reason other than technical limitation why experiments should not be performed involving six or a dozen directions. In Figure 4-3 a minimal model is defined for an experiment involving six directions. This minimal model is constructed in the spirit of Fine with successive analyzer positions ($A_1B_1A_2B_2A_3B_3$) separated by the same angle θ, for various values of θ, a uniform density $\rho(\lambda) = 1/9$ (implicit in Figure 4-3 is $\Lambda = [0,9]$), and disjoint categories for non-commuting observables. Once again, the argument of a function on an interval $(n, n+1)$ is $\lambda - n$. Furthermore, $g\theta''$ is defined as $Q'' = 1/2 \cos^2 5\theta$. Single rejection rates for the case of 6 directions are 66.7% and joint A_iB_j-rejection rates are 88.9%. In general, for n directions (n even) an $(n/2)^2 \times n$ grid will be required with single rejection rates of $1-2/n$ and joint rejection rates of $1-(2/n)^2$, both of which tend to 1 (i.e., 100%) as $n \to \infty$. There is no reason to suppose that experimentally encountered rejection rates (which will be worse when mechanical inefficiencies are allowed for) will behave appropriately. Minimal models do not extend well to straightforward extrapolations of Bell-type experiments.

FIGURE 4-3 The Minimal Model: N=6.

A_1	A_2	A_3	B_1	B_2	B_3
f	▨	▨	g_θ	▨	▨
f	▨	▨	▨	$g_{\theta'}$	▨
f	▨	▨	▨	▨	$g_{\theta''}$
▨	f	▨	g_θ	▨	▨
▨	f	▨	▨	g_θ	▨
▨	f	▨	▨	▨	$g_{\theta'}$
▨	▨	f	$g_{\theta'}$	▨	▨
▨	▨	f	▨	g_θ	▨
▨	▨	f	▨	▨	g_θ

As Sharp and Shanks[30] have noted, the trouble here goes beyond the failure of Fine's analysis to provide satisfactory models for new experiments. There is a failure to provide conceptually adequate models for current experiments (even if there is a certain numerical adequacy). The λ which are to determine particle responses to various orientations of the analyzers are set at the source. It is peculiar that the λ determine responses only for current pairs of analyzer settings in each wing--but not for other potential analyzer settings which might be realized while the particles are in flight. To deny that it is determined at the source how a particle will respond to a distant analyzer in any one of non-denumerably many positions is tantamount to denying that a local deterministic account of spin correlation statistics has been given.[31] The other horn of the "prismers dilemma" for minimal models is to have none of the particles show up--since rejection rates would have to be 100% if responses to all possible settings are to be determined at the source.

To calculate the rejection rates for maximal models in which $n > 4$, one must first decide what the appropriate extrapolations are. According to Fine,[32] the guiding idea behind maximal models is that three of the four response functions can be defined for any λ, but not all four. If we were to take it that each λ is unresponsive to exactly one analyzer setting and that for each setting there is some unresponsive λ--which is one way to extrapolate the tiling pattern in Figure 4-2--then we would get, for the case of n settings, an nxn grid with, (for example) all squares on one diagonal blocked out. Not only could one not use Fine's f and gθ functions in this case, alas there is a further difficulty. For any group of four response functions, the single and joint distributions (including $P_{A_iA_kB_jB_l}$) generated would be related as in classical random variable theory on every interval [m, m+1] except for the four intervals on which one of the response functions is blocked out. These distributions would satisfy (SCH) except on four of the n equally weighted intervals. Thus, while the single and A_iB_j-joint distributions would have associated rejection rates that tend to 0 as $n \to \infty$, violations of (SCH) for any given quadruple $[A_iA_kB_jB_l]$ of analyzer settings occur only on a subset of comprising four out of n intervals--a subset that tends to a set of measure 0 as $n \to \infty$. This way of extrapolating maximal models cannot be right, as in the limit it provides models satisfying (SCH)--the very inequality that prism models were to subvert.

Happily there is a general way of building models for the extrapolated cases that employs Fine's f and gθ functions. The models for n = 6 and n = 8 are reproduced in Figure 4-4.[33] In this case an nxn grid is required, with single rejection rates of 1/2-1/n and joint rejection rates of 1-2/n, tending to 50% and 100% respectively, as $n \to \infty$. The latter of these rejection rates is clearly unacceptable. Thus it seems that maximal models, like minimal models, will fail to provide adequate models for very natural extrapolations of the sorts of experiments used to test the Bell-type inequalities.[34]

SECTION FOUR: SYNCHRONIZATION MODELS

The synchronization model approach[35] differs from the previous two approaches to the modelling of the spin correlation measurement statistics in that it employs the random variable/phase space apparatus. The synchronization model approach will be criticized

FIGURE 4-4 The Maximal Model: N=6, N=8.

A_1	A_2	A_3	B_1	B_2	B_3
f	▨	▨	g_θ	$g_{\theta'}$	$g_{\theta''}$
▨	f	▨	g_θ	g_θ	$g_{\theta'}$
▨	▨	f	$g_{\theta'}$	g_θ	g_θ
g_θ	g_θ	$g_{\theta'}$	f	▨	▨
$g_{\theta'}$	g_θ	g_θ	▨	f	▨
$g_{\theta''}$	$g_{\theta'}$	g_θ	▨	▨	f

A_1	A_2	A_3	A_4	B_1	B_2	B_3	B_4
f	▨	▨	▨	g_θ	$g_{\theta'}$	$g_{\theta''}$	$g_{\theta'''}$
▨	f	▨	▨	g_θ	g_θ	$g_{\theta'}$	$g_{\theta''}$
▨	▨	f	▨	$g_{\theta'}$	g_θ	g_θ	$g_{\theta'}$
▨	▨	▨	f	$g_{\theta''}$	$g_{\theta'}$	g_θ	g_θ
g_θ	g_θ	$g_{\theta'}$	$g_{\theta''}$	f	▨	▨	▨
$g_{\theta'}$	g_θ	g_θ	$g_{\theta'}$	▨	f	▨	▨
$g_{\theta''}$	$g_{\theta'}$	g_θ	g_θ	▨	▨	f	▨
$g_{\theta'''}$	$g_{\theta''}$	$g_{\theta'}$	g_θ	▨	▨	▨	f

along two lines: it will be argued that the approach constitutes an ad hoc solution to the problem at hand, and it will also be argued that the postulated particle retardation effects are unacceptable for experiments which have already been performed.

The assumption on which synchronization models lie is that in a Bell-type experiment particles may be delayed differently in passing through their respective analyzers or in the subsequent journey to the detectors so as to be significantly retarded relative to one another, and hence fail to produce counts that are in coincidence.[36] When such retardation occurs, particles are said to be out of synchronization. Inasmuch as the probabilities in correlation experiments are analyzed in terms of coincidence count rates, this possibility clearly rates some discussion. In the prism model approach the coincidence count rates were to be such that for any A_iB_j-experiment--even an ideal one--certain particles would fail to show up. In the synchronization model approach, all the particles can show up, but for any A_iB_j-experiment not all of those that show up will be in synchronization.

Fine proceeds by introducing a coefficient of synchronization as follows:

1) The coefficient of synchronization $C_{ij} =_{df}$ the probability that a pair of particles in classically determinate state λ will give rise to a coincidence count when the A- and B-analyzers are in their i^{th} and j^{th} positions respectively, given that both particles do in fact pass their respective analyzers.

Fine supposes that response functions A_i and B_j are defined for all $\lambda \in \Lambda$, distributed uniformly on [0,1]. He then considers the following probabilities:

2) (i) $P_A(\lambda,i) =_{df}$ the probability for an A-wing count for a particle in state λ when the A-analyzer is set in its i^{th} position. ($P_B(\lambda,j)$ is similarly defined).

(ii) $P_{AB}(\lambda,i,j) =_{df}$ the probability for a coincidence count when a pair of particles in state λ is emitted and the A- and B-analyzers are in their i^{th} and j^{th} positions respectively.

Let $P_A(\lambda,i) = A_i(\lambda)$ and let $P_B(\lambda,j) = B_j(\lambda)$. In a synchronization model:

3) $P_{AB}(\lambda,i,j) = A_i(\lambda)\cdot B_j(\lambda)\cdot C_{ij}(\lambda)$, where $C_{ij}(\lambda) \neq 1$. Such a model must satisfy:

4) $$P_{A_i} = \int_0^1 A_i(\lambda)\rho(\lambda)d\lambda = 1/2$$

5) $$P_{B_j} = \int_0^1 B_j(\lambda)\rho(\lambda)d\lambda = 1/2$$

6) $$P_{A_iB_j} = \int_0^1 A_i(\lambda)B_j(\lambda)C_{ij}(\lambda)\rho(\lambda)d\lambda = Q \text{ or } Q'$$

In Fine's construction of synchronization models, some of the devices used in the construction of prism models are re-deployed. From Chapter Three it will be recalled that $Q = 1/2\cos^2\theta$ and $Q' = 1/2\cos^2 3\theta$. Further, the functions f, $g\theta$ and $g\theta'$ are to be understood as they were defined in the previous section in connection with prism models. Using these functions we know that:

7) $\int_0^1 f(\lambda)d\lambda = 1/2$, $\int_0^1 g\theta(\lambda)d\lambda = 1/2$,

$\int_0^1 g\theta'(\lambda)d\lambda = 1/2$, $\int_0^1 f(\lambda)g\theta(\lambda)d\lambda = Q$

$\int_0^1 f(\lambda)g\theta'(\lambda)d\lambda = Q'$

In Fine's view, we must consider two cases:

Case 1: $Q' < Q$

Let $A_i(\lambda) = f(\lambda)$ for all $i = 1,2$

$B_j(\lambda) = g\theta(\lambda)$ for all $j = 1,2$

$C_{11}(\lambda) = C_{21}(\lambda) = C_{22}(\lambda) = 1$, for all λ

$C_{12}(\lambda) = g\theta'(\lambda)$ for all λ.

Then:

a) $\int_0^1 A_i(\lambda)B_j(\lambda)C_{ij}(\lambda)d\lambda$

$= \int_0^1 f(\lambda)g\theta(\lambda)d\lambda = Q$

for $C_{ij} = (1,1), (2,1), (2,2)$

And,

b) $\int_0^1 A_1(\lambda)B_2(\lambda)C_{12}(\lambda)d\lambda$

$= \int_0^1 f(\lambda)g\theta(\lambda)g\theta'(\lambda)d\lambda$

$= \int_0^1 f(\lambda)g\theta'(\lambda)d\lambda = Q'$

Case 2 $Q < Q'$

Let $A_i(\lambda) = f(\lambda)$, $c = 1,2$

$B_j(\lambda) = g\theta'(\lambda)$, $j = 1,2$

$C_{11}(\lambda) = C_{21}(\lambda) = C_{22}(\lambda) = g\theta(\lambda)$ for all λ

$C_{12}(\lambda) = 1$ for all λ

Then:

a) $$\int_0^1 A_i(\lambda)B_j(\lambda)C_{ij}(\lambda)d\lambda$$

$$= \int_0^1 f(\lambda)g\theta'(\lambda)g\theta(\lambda)d\lambda$$

$$= \int_0^1 f(\lambda)g\theta(\lambda)d\lambda = Q$$

for $C_{ij} = (1,1), (2,1), (2,2)$. And:

b) $$\int_0^1 A_1(\lambda)B_2(\lambda)C_{12}(\lambda)d\lambda$$

$$= \int_0^1 f(\lambda)g\theta'(\lambda)d\lambda = Q'$$

In the following discussion of Fine's synchronization models, I will discuss Case 1. My comments will apply equally well to Case 2.

The first thing to notice is that the quantum mechanical probabilities are angle-dependent correlations:

1) $P(A_iB_j) = Q = 1/2 \cos^2\theta$,

for $(i,j) = (1,1), (2,1), (2,2)$

2) $P(A_1B_2) = Q' = 1/2 \cos^2 3\theta$.

Thinking in terms of apparatus consisting of variously oriented Stern-Gerlach magnets, we notice that in a Case 1 synchronization model, the correlations for (i,j) = (1,1), (2,1), (2,2) are analyzer-dependent, since for these settings, $C_{ij} = 1$. Being analyzer dependent means that they are correlations which depend solely on the orientations of the analyzer. However, for (i,j)=(1,2) the correlations are retardation-dependent. That is, they depend not only on the orientations of the analyzers but also upon apparatus retardation effects. The comparison of the (1,2)-type correlations with the (1,1), (2,1), (2,2)-type correlations is a bit like the comparison of apples and oranges: they are different types of correlation, (whereas the original quantum correlations are all of the same analyzer-dependent type), notwithstanding the fact that the numbers may come out right.

If we continue to think of measuring spin components using Stern-Gerlach magnets, we might ask why some particles get delayed so as not to arrive in coincidence in a (1,2)-experiment whereas when the apparatus is oriented to measure say (1,1) it is certain that particles which emerge from the magnets arrive in coincidence at the detectors. The only difference between the two cases is an angle between variously oriented spacelike separated magnets. From this point of view, synchronization models are merely ad hoc. We have no theoretical grounds for supposing that the apparatus will delay particles in the journey through the analyzer to the detectors when some experiments are performed but not when other experiments of a similar kind are performed. There is likely very little evidence of these delay effects either.

Another point, related to this last point above, is this: if we make the usual assumption of system-apparatus independence so that the states of systems are independent of the orientations of the magnets and vice-versa, then there seems to be an element of non-locality to the synchronization model approach. A (1,1)-experiment (involving no retardation effects) differs from a (1,2)-experiment (which involves retardation effects), by the setting of the analyzer in the B-wing. Assume that the setting of the apparatus (the A-wing and B-wing orientation) was not arranged until the last possible moment (so that any communication between the analyzers would have to be superluminal if it was to affect measurement outcomes). It is difficult to see how, short of there being non-local effects, (Einstein non-local effects, that is), the A-wing apparatus could 'know' that a B_2- rather than a B_1-experiment was being performed in the B-wing, and similarly, how the B-wing apparatus could 'know' that an A_1 rather than an A_2 experiment was being performed in the A-wing. In short, it is difficult to see how the apparatus, (consisting of an A-analyzer spacelike separated from a B-analyzer), could 'know' when to retard particles and when not to retard particles--unless, of course, Einstein locality was violated.

There is a further important difficulty for the synchronization models approach. Most of the experiments to test Bell's theorem, performed to date, have been concerned with photon polarization correlation statistics. These are measurement statistics generated by pairs of photons in some appropriate singlet state. Photons do not exist at velocities less than the speed of light. It is consequently difficult to see how the apparatus (where the equipment in the A-wing is essentially the same as the equipment in the B-wing) could delay a photon so that it could be detected as being out of synchronization with its mate. Synchronization models, even if they yield the right numbers, do so in a way that we have theoretical grounds for supposing to be manifestly implausible, given the sort of physical system currently under analysis in experimental tests of Bells Theorem.

SECTION FIVE: CONCLUSION

In this Chapter I have considered three proposals of Arthur Fine for adjusting classical probability theory and/or the way in which it is applied to the statistics generated by Bell-type correlation experiments. All of Fine's proposals are beset by grave difficulties. Therefore I do not believe that any of his proposals can constitute a viable analysis of what is going on in Bell-type correlation experiments.

We seem to be left with a least three options: (a) find some other ways of adjusting classical probability theory and/or the way we apply it; (b) abandon classical probability theory in favour of some non-classical probability theory; and (c) stay within the framework of classical probability theory.

One matter to be discussed in the next Chapter is the possibility of staying within the framework of classical probability theory but of denying the force of Bell's theorem by attacking Bell locality. If there are circumstances under which Einstein locality does not imply Bell locality, then in those circumstances we might use the random variable/phase space apparatus to retrieve the quantum spin correlation measurement statistics while not violating Einstein locality.[37]

FOOTNOTES

[1] Stairs (1978), p. 91.

[2] In the discussion of the joint distribution issue in Chapter One, it was seen that it was a good question as to whether we can interpolate joint distributions for non-commuting quantities. There are, of course, proofs that we cannot interpolate joint distributions, but it was also seen in Chapter One that those proofs rely on assumptions which a hidden variables theorist might well reject.

[3] Fine (1974), (1981), (1982a), (1982b), (1982c), (1986).

[4] Fine (1974). See also Stairs (1979).

[5] The rules of quantum probability theory differ from phase space rules (as quantum mechanics is standardly interpreted).

[6] Fine (1974), p. 282.

[7] Fine (1982c), p. 1309.

[8] Ibid.

[9] Fine (1974), p. 283.

[10] Fine (1982c), p. 1310.

[11] Ibid., p. 1310.

[12] The SRV approach attacks clause (b) of the definition of Bell locality. See Chapter Three.

[13] Fine (1981), p. 544.

[14] For the assumption of system-apparatus independence see the discussion of Bell locality in Chapter Three.

[15] Cartwright (1983), pp. 177-178.

[16] Ibid., p. 178.

[17] Fine (1981), (1982a), (1982b).

[18] Fine (1986).

[19] Sharp and Shanks (1985).

[20] See Fine (1981), (1982a).

[21] Fine (1982a), p. 282. Slightly corrected.

[22] Ibid., p. 283. Slightly corrected.

[23] Ibid., pp. 284-286.

[24] That is, $\rho(\lambda) = 1/4$.

[25] Fine (1982a), p. 286.

[26] Ibid., p. 280.

[27] Experiments where four observables are discussed and where there are uniform angular separations.

[28] Fine (1986), p. 53.

[29] These rejection rates can be seen by examining the shaded areas in Figure 4-1.

[30] Sharp and Shanks (1985), p. 553.

[31] In any case, there is no reason to believe that all current experiments, with two settings in each wing, have measured exactly the same spin components (or polarization components). So unless the λ which determine responses to the four settings vary from laboratory to laboratory, responses must be regarded as being determined for a sufficient range of orientations of the analyzers as to render the minimal model untenable--even given experiments which are 20% efficient.

[32] Fine (1982a), p. 287.

[33] Note that $g\theta'''$ is defined using $Q''' = 1/2 \cos^2 7\theta$.

[34] A number of other defects in the prism model programme are discussed in Sharp and Shanks (1985).

[35] See Fine (1981), (1987a).

[36] From an experimental point of view, the data in correlation experiments comes in the form of coincidence count rates for various orientations of the analyzers.

[37] Any such approach would naturally violate Bell locality, but it is Einstein locality which is the locality condition crucial to physics.

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CHAPTER FIVE

BELL'S THEOREM AND THE REALIST/ANTI-REALIST DEBATE

> When they argue for their position, realists typically argue against some version of idealism--in our time, this would be positivism or operationalism . . . And the typical realist argument against idealism is that it makes the success of science a miracle.
>
> H. Putnam[1]

SECTION ONE: INTRODUCTION

The second of the three questions which concern this study is: what is the relevance of Bell's theorem to the realist/anti-realist debate? In Chapter Three it was shown how Bell's theorem rules out Bell local hidden variables theories of quantum spin correlation measurement statistics. In the literature, however, one often finds it claimed that Bell's theorem, and the evidence which supports it, rules out local realism.[2]

In this Chapter this claim will be analyzed and subjected to a range of criticisms. In the first stage of the analysis it will be agreed that Bell's theorem is relevant to the assessment of certain realist theories. But it will also be argued that there are certain forms of realism which are Bell local but nonetheless immune to the force of Bell's theorem. Further, it will be argued that Bell's theorem rules out certain anti-realist theories as well. To the best of my knowledge, this suggestion has not been made before. There are, however, anti-realist theories which are wholly immune to Bell-type considerations.[3]

It will emerge that a crucial concept--one which all Bell-susceptible theories[4] have in common, be they realist or anti-realist --is the concept of classical determinacy. Thus there is a sense in which the Bell issue may cut across the traditional lines drawn in the realist/anti-realist debate.

In the second stage of the analysis attention will be given to van Fraassen's critique of epistemic realism. In this part of the discussion the assumption of Bell locality--though not the assumption of Einstein locality--will be criticized. Epistemic realism, in the context of super-deterministic physics,[5] will be defended against van Fraassen's critique.

The aim of this Chapter, then, is to establish four main points: (1) that the claim that Bell's theorem rules out local realism is not wholly accurate[6]; (2) that certain anti-realist theories can be susceptible to Bell-type considerations; (3) that one important local realist response to Bell-type arguments calls into question Bell locality and not Einstein locality, and (4) (arising from the above) that it is possible to opt for the classical determinacy assumption underlying the Bell argument, rather than, as is commonplace, the assumption of Bell locality. Point (4) is important since theorists who have opted for the hidden variables assumption when faced with Bell's dilemma have also felt constrained to embrace some form of Einstein non-locality. It is an important conclusion of this Chapter that one can be a hidden variables theorist and yet not embrace violations of Einstein locality.

SECTION TWO: REALISM AND CLASSICAL DETERMINACY

It has already been suggested in this study that one response to the puzzling Bell-type statistics is the adoption of Bohr's anti-realism, i.e., instrumentalism. On such an anti-realist view, quantum mechanics is assessed merely in terms of the correctness of its predictions. As to the character of the mechanisms which underlie measurement results in quantum mechanics, the Bohrian view advises, 'Don't ask.' On such a view it is not that quantum mechanics is an incompetent explainer, it is that one cannot properly ask for explanations of statistical results. In Bohr's terms, it is illegitimate to speak of observables taking values outside of the context of experiments to determine those values. In the Bell case, all that matters is that the Bell-type spin correlation measurement statistics are correctly predicted by quantum mechanics.

The traditional opponents of anti-realism (and instrumentalism) are the realists. According to common lore, however, on the assumption of Bell locality one cannot be a realist about the Bell-type measurement statistics. In the remainder of this section I consider some forms of realism which are susceptible to Bell-type considerations. Before doing so, however, it will be important to make some distinctions which will turn out to be useful when clarifying realist options.

The first distinction to be made is between classically determinate states λ_i and classically determinate states λ_m. The states λ_i can be thought of as lists of measurement-independent spin component values. The states λ_m can be thought of as lists of spin component values that would be found were certain measurements to be performed.

The second distinction to be made is between Ival-sentences and Mval-sentences. The distinction may be drawn as follows:

(Ival) '$\mathrm{Ival}(A)_{\emptyset} \in M$' means 'In quantum state $\emptyset$ the measurement-independent value of spin component A lies in Borel set M.'

(Mval) '$\mathrm{Mval}(A)_{\emptyset} \in M$' means 'In quantum state $\emptyset$ the value that would be found upon measurement of spin component A lies in Borel set M.'

The states λ_i provide truth conditions for all Ival-sentences and the states λ_m provide truth conditions for all Mval-sentences.

In this section it will be argued that there are several realist routes to classically determinate states λ which will get entangled in Bell-type difficulties. First of all it is important to consider some formulations of realism.

Devitt[7] has defined the scientific realist position as follows: scientific realism is the theory according to which science is committed to the unobservable entities it postulates. Again, in a related vein, Fine has summarized Einstein's view of realism as follows:

> If we look at Einstein's remarks on realism, I think that the basic idea surfaces right away . . . 'Physics is an attempt conceptually to grasp reality as it is thought to be independently of its being observed. In this sense one speaks of "Physical reality".'
>
> The key realist idea here is that the conceptual model (or theory) is to be understood as an attempt to treat things as we imagine they would be were they not being observed . . .[8]

Or as Einstein put it himself in 1955:

> It is basic for physics that one assumes a real world existing independently from any act of perception. But this we do not know. We take it only as a programme in our scientific endeavours. This programme is, of course, prescientific and our ordinary language is already based on it.[9]

There is no mention of classically determinate states in any of these quotations. Is Bell's theorem relevant to the assessment of Devitt's or Einstein's realism?

One way in which Bell's Theorem can be relevant is as follows: Let the observer-independent reality (or the unobservable entities postulated by science) have classically determinate spin component values. These values may, for any pair of particles in a Bell-type experiment, be given in the form of a list $\lambda_i \in \Lambda_i$. Assume further a 'no disturbance' principle according to which measurement reveals pre-existing or measurement-independent spin component values. In this case the measurement-independent spin component values *just are* the spin component values that would be found were measurements to be performed, i.e., $\lambda_i = \lambda_m$. In this case we will simply refer to states λ. Given the quantum prediction $A(\vec{a},\lambda) = -B(\vec{a},\lambda)$, we know from Bell's theorem that there is no probability density $\rho(\lambda)$ which is capable of returning the quantum spin correlation measurement statistics.

That the observer-independent realm might be classically determinate in some way such as the above was a conclusion strongly indicated by EPR-type considerations. In the analysis of the spin-version of the EPR argument presented in Chapter Three, it was seen that one way to explain the pattern of spin component measurement results was to view the quantum singlet state as an incomplete description of any pair-system. Quantum predictions and Einstein locality implied the EPR-type conclusion that measurement results were determinate for any spin component. Since one could predict the result of measuring any spin component with certainty without in any way interfering with the system, one could apply the EPR reality criterion to conclude that all spin components took values independently of measurement. The Bell result is, as noted, relevant to this position because we can index these values by states $\lambda \in \Lambda$ . . .

Einsteinian realism deserves to be called classical ontological realism because it views the denizens of the microcosm--inasmuch as they play a role in Bell-type experiments--as being classically determinate entities.

A related route to classical determinacy of spin components for the systems in a Bell-type experiment derives from the realist idea of the mind-independence of reality. In addition to Bohr's instrumentalism, idealism has been an enduring strand in work on the foundations of quantum mechanics.[10] According to the von Neumann/Wigner view, measurement interactions are distinct from other physical interactions because they involve a conscious observer whose intervention explains the collapse of the wave packet. Consider the singlet state Ψ_0:

1) $\Psi_0 = 1/\sqrt{2}\,[\phi_1(+1)\phi_2(-1)-\phi_1(-1)\phi_2(+1)]$.

At the time of measurement a conscious observer intervenes and becomes, for example:

2) $\phi_1(+1)\phi_2(-1)$,

corresponding to spin +1 along $\vec{a}$ on particle 1 and spin -1 along $\vec{a}$ on particle 2.

The mind-independent realist wants to remove the observer from the stage, so to speak, and put him or her back into the audience where he or she classically belongs. Such a realist will view measurement interactions as physical interactions like any other physical interactions. On this view, the reason one gets spin component values upon measurement is that those observables take values independently of measurement and the operations of consciousness. Once again, observer-independent spin component values (revealed by measurement) will be indexed by $\lambda \in \Lambda$ . . .

Thus we can see that there are several realist motivations for the assumption of classical determinacy for spin component values in just the way required to get the Bell argument going.

To someone dissatisfied with Bohr's philosophy and its hostility to explanation, the possibility remains that there is some natural explanation of the puzzling correlation phenomena. This explanation in turn may involve recourse to unobserved entities with certain properties capable of explaining the statistics. What one might call explanatory realism is the view that one is committed to the existence of the entities (and their properties) which one uses in explanations. A special case of explanatory realism is causal explanatory realism. Nancy Cartwright is a representative of this view. In her discussion of the connection between explanation and truth, she suggests that, at least for causal explanations, if x explains y, and y is true then x should be true as well. She comments:

> Suppose we describe the concrete causal processes by which a phenomenon is brought about. That kind of explanation succeeds only if the process described actually occurs. To the extent that we find the causal explanation acceptable, we must believe in the causes described.[11]

If the causal explanation of the Bell-type spin correlation measurement statistics proceeds by recourse to entities which are classically determinate with respect to spin component values (these values being revealed by measurement), then on the assumption of Bell locality the viability of the explanation will be undermined by the Bell argument.

Cartwright's position, as it stands, is not quite realist, for nothing has been said about the character of the causes, and in particular their mind-independence or observer-independence. The position becomes fully a realist position if we assume that the causes x which explain the phenomena y are mind-independent.

From the discussion of determinism in Chapter One, it can be seen immediately that a Laplace-deterministic causal account of the Bell-type correlation statistics would require classically determinate and complete states $\lambda \in \Lambda$. Of course, causal explanations do not have to be deterministic--they can be stochastic. For instance, one would naturally anticipate that a stochastic electrodynamical account of the spin correlation measurement statistics would not be deterministic. However, since the theory boasts a classically determinate ontology one would expect the Bell result to be relevant to the assessment of the resultant explanation.

It is certainly most important to note that the determinism/indeterminism issue is not the same as the realist/anti-realist issue. Neither is the causal/acausal explanation debate the same as the debate between realists and anti-realists. The position of causal explanatory realism is a realist theory because of the mind- or observer-independence of the causal commitments and not because of the causal commitments themselves.

The scientific realist position has been variously formulated. Some of its formulations seem to involve classical determinacy in precisely the sense required by the Bell argument. For instance, MacIntosh[12] defines scientific realism as follows: statements which express scientific theories have literal and determinate meaning given by their truth conditions, their satisfaction making the theory true (otherwise false) and these conditions must involve only states of an objective and independent world.

In what follows I will assume that an 'objective and independent world' is a mind-independent world, in order to bring this semantic formulation of realism in line with the earlier discussion of that view.

Consider the theoretical sentences of quantum mechanics. These are Val-sentences of the form:

1) $\mathrm{Val}(A)_{\emptyset} \in M$,

(to be read 'In quantum state $\emptyset$ observable A takes a value in Borel set M).[13] As noted by Fine,[14] quantum mechanics provides only the following truth conditions for Val-sentences:

(T_{qm}) "$\mathrm{Val}(A)_{\emptyset} \in M$" is true if the state of the system is a superposition of A-eigenstates, each with an eigenvalue in the Borel set M.

The problem is that there are states of systems in quantum mechanics (general linear superpositions of A-eigenstates) for which '$\mathrm{Val}(A)_{\emptyset} \in M$' has no quantum mechanically determined truth conditions --although a probability is always determined. In this way quantum states underdetermine truth conditions for theoretical sentences. If quantum mechanics is unsupplemented with hidden variables (to determine values for all observables) then there will be sentences of quantum mechanics which will not have a literal and determinate meaning by the MacIntosh criterion--because their truth conditions are undetermined. Thus, quantum mechanics minus hidden variables would not be judged a realist theory by MacIntosh.

Suppose now that we attempt to make quantum mechanics a MacIntosh-realist theory by introducing hidden variables to determine the truth conditions for all spin component value sentences. We will proceed in some way as follows: with each quantum state $\emptyset$ there will be associated a phase space Λ of classically determinate states λ, and each quantum observable A will be associated with a random variable f_A on Λ.[15] The set $[f_A(\lambda):\lambda \in \Lambda]$ will be the possible values of A in state $\emptyset$. The requisite truth conditions will be stated as follows:

(T_{hv}) "$\mathrm{Val}(A)_{\emptyset} \in M$" is true at classically determinate state λ iff $f_A(\lambda) \in M$.

And it comes as no great surprise to discover that when this is done for spin observables, (and where it is, of course, assumed that the result of measuring a spin component reveals the measurement-independent value), we have precisely introduced the random variable/phase space apparatus required to get the Bell argument going.

So far I have examined the realist positions which commit one to a mind-independent classically determinate world. This view is manifestly threatened by the Bell result on the assumption that measurements reveal the pre-existing or measurement-independent spin component values to us.

There are at least two other realist options which appear to be threatened by the Bell result--and are ruled out on the assumption of Bell locality.[16] The first of these realist theories states that there is an observer-independent classically determinate world. (So there are states λ_i which provide for a statement of the truth conditions for all Ival-sentences.[17]) Measurements, according to this theory, however, do not always reveal the pre-existing or mind-independent spin component values. There are, on this view, states λ_m which provide for a statement of the truth conditions for all Mval-sentences. The states $\lambda_m \in \Lambda_m$ are nothing more than lists of spin component values which would be found were measurements performed. Given the quantum prediction $A(\vec{a},\lambda) = -B(\vec{a},\lambda)$, we know from the Bell result that there is no probability density $\rho(\lambda_m)$ on Λ_m which will return the quantum spin correlation measurement statistics.

The second realist theory states that the measurement- or mind-independent world is not classically determinate as regards spin component values--so spin components, independently of measurement, do not take values (thus truth conditions are undetermined for all Ival-sentences).[18] Measurements, according to this theory, do not yield or reveal pre-existing or mind-independent spin component values--since there are none to reveal. Nevertheless, according to this theory, measurement results are classically determinate in the sense that there are states $\lambda_m \in \Lambda_m$ which are nothing more than lists of values which we would find were certain measurements performed. On this view, notwithstanding the fact that truth conditions are undetermined for the Ival-sentences, all Mval-sentences have λ_m-determined truth conditions. Once again, we know that there is no probability density $\rho(\lambda_m)$ which will return the spin correlation measurement statistics.

There are thus a number of realist routes to the sort of classical determinacy needed to get the Bell argument going. Whether motivated by mind-independence, explanatory concerns or semantic considerations, there is a clear sense in which we can say that some realist theories, on the assumption of Bell locality, are ruled out by Bell's theorem.

SECTION THREE: REALISM WITHOUT CLASSICAL DETERMINACY

Could there be realist theories which do not imply the kind of classical determinacy on which the Bell argument is based? From a conceptual point of view it would appear that there are two options to consider in breaking the traditional links between realism and classical determinacy in such a way as to avoid the force of the Bell argument.[19]

The first option is the position of the inaccessible realist. Here the Ival-sentences all have truth conditions determined by states $\lambda_i \in \Lambda_i$. These measurement-independent spin component values are inaccessible in the sense that they are never revealed by measurement--though we know that the values cannot be anti-correlated in accord with quantum mechanics for arbitrary directions $\vec{a}$. What is crucial to the present theory is that the Mval-sentences have undetermined truth conditions--there are no states $\lambda_m \in \Lambda_m$!

The second option is the position of the indeterminate realist. Here both the Ival and Mval-sentences have undetermined truth conditions. Again we could think of the systems in a Bell-type experiment as being particle-pairs, each particle having simultaneous exact values for position and momentum but being classically indeterminate as regards measurement-independent and measurement-dependent spin component values. That is, there are no states $\lambda_i \in \Lambda_i$ or $\lambda_m \in \Lambda_m$.

Both of these realist options avoid the Bell hidden variables assumption and so do not violate Bell locality. They are, however, subject to a certain difficulty which I shall call the simple anti-correlation argument: when spin component measurements are performed--e.g., in the A-wing[20] along the direction $\vec{a}$, we always find a value for spin along $\vec{a}$ (±1). When such a measurement is performed in the A-wing the measurement result for spin along $\vec{a}$ becomes determinate in the B-wing--so if $S_1(\vec{a}) = +1$ then $S_2(\vec{a}) = -1$ and so on.

In the spirit of realism, let us assume that there are no crafty interventions of consciousness or other effects of "mind over matter". The trouble is that we have foresworn states λ_m and so we must now explain how we get a value--+1 say--for spin along $\vec{a}$ on particle 1 when no measurement results are determinate. One possibility is that spin along $\vec{a}$ is a relational property between a micro-system (a spin - 1/2 particle, for instance) and a macro-system (a Stern-Gerlach magnet apparatus, for example). On this view, when micro-system and macro-system are appropriately related, then spin along $\vec{a}$ will take one of its eigenvalues. When the relationship ceases to obtain, then spin along $\vec{a}$ ceases to take a value.[21]

Assume further that we do not decide to orient the Stern-Gerlach magnet for a $S_1(\vec{a})$-measurement until the last possible moment so that there can be no luminal or subluminal communication between the apparatus in the B-wing and the apparatus in the A-wing. Then when the measurement result $S_1(\vec{a})$ becomes determinate (showing +1, for example), then at that time the measurement result--the value that would be found were an $S_2(\vec{a})$-measurement to be performed--becomes determinate, (showing -1 in this case). The question before us is this: how can the fact that a measurement result has become determinate in the A-wing make a previously indeterminate measurement result determinate, for a spacelike separated system in the B-wing?

Besides cosmic accidents, one suggestion which naturally springs to mind is that there are non-local influences in operation: the measurement result in the B-wing becomes determinate instantaneously because of what is done to the A-wing particle as the result of a non-local influence.

It is, perhaps, a good question as to whether this non-locality is Einstein non-locality. What can be said is this: what has become determinate in the B-wing as the result of measurement in the A-wing, is the value that would be found were an $S_2(\vec{a})$-measurement to be performed. The non-local disturbance is thus a disturbance of a dispositional property of the B-wing system. We can put things this way: in virtue of the fact that $S_1(\vec{a}) = +1$, the Mval-sentence,

'$Mval(S_2(\overleftarrow{a}))_0 = -1$', which prior to the $S_1(\overleftarrow{a})$-measurement had undetermined truth conditions, now has determined truth conditions. Measurement on a system spacelike separated from the B-wing system can render determined previously undetermined truth conditions for a sentence concerning the B-wing system. This is highly surprising, for prior to measurement of $S_1(\overleftarrow{a})$, it is not merely that we are ignorant of the truth conditions for the B-wing sentence. Rather, the B-wing sentence did not, as a matter of fact, even have determined truth conditions. There seems to be an element of non-local semantics here at the very least!

Realist theories which do not violate Bell locality nevertheless seem to be caught up in a kind of non-locality when they come to explain the simple facts of anti-correlation of spin component measurement results predicted by quantum mechanics. So while one can be a realist in the face of the Bell argument, the benefits seem to be somewhat questionable. Of course there are realist theories which violate both Bell locality and Einstein locality. Later in this Chapter, I will explore the possibility of there being a realist theory of the spin component measurement statistics which while being Einstein local is nevertheless not Bell local.

SECTION FOUR: ANTI-REALISM AND THE BELL THEOREM

So far I have examined some of the implications of Bell's theorem for realistic theories of the spin component measurement statistics. It will be argued here that Bell's theorem has implications for certain anti-realistic theories of these same measurement statistics.

One important kind of anti-realism is fictionalism. On this view, the unobserved entities appealed to in explanations of phenomena literally do not exist--rather, they are fictions. The idea is an old one. As Vaihinger, a classical proponent of fictionalism put it:

> An idea whose theoretical untruth or incorrectness, and therewith its falsity, is admitted, is not for that reason practically valueless and useless; for such an idea, in spite of its theoretical nullity may have great practical importance.[22]

In the version of fictionalism relevant to quantum mechanics we might say that microphysical object statements are abbreviated ways of saying that certain phenomena are (or appear to us) as if certain microphysical objects with certain properties existed, but in fact no such microphysical objects do exist. Electrons and their ilk are thus fictions--mental constructs or whatever.

These fiction-statements have the form 'x is as if p', where what goes in for 'x' is a description of observable phenomena or behaviour, and what goes in for 'p' is a proposition asserting the existence of certain microphysical objects with certain properties (or certain types of microphysical object with certain types of property). For example: 'The tracks in the bubble chamber are as if there are particles like billiard balls writ small', 'The flash on the screen is as if there is an electron.' Clearly an 'as if' statement may be true even though what goes in for p may be false. As Henry Price has noted:

> It is clear that the whole of natural science . . . could be understood on this 'as if' basis so long as our sense experiences are sufficiently complex and detailed to enable us to distinguish between those sense-impressions which are as if one sort of material object existed, and those which are as if another sort of material object existed.[23]

Consider the following 'as if' statement: 'The Bell-type correlation phenomena are (or appear to us) as if there were pairs of classically determinate particles'. In this case the phenomena are the spin correlation measurement statistics. The fictions which are to do the explaining are pairs of classically determinate particles with appropriately correlated spin component values. On the assumption of Bell locality we know there is no distribution $\rho(\lambda)$ over the fictional spin component values which can return the spin correlation measurement statistics. Thus, on the assumption of Bell locality, pairs of classically determinate particles are highly inconvenient fictions--they cannot explain the phenomena. On the assumption of Bell locality certain fictionalist options are shown to be unacceptable.

When Bell stated his theorem in 1964, he did not discuss the claim that measurements reveal to us pre-existing or measurement-independent spin component values. Rather, Bell assumed that it was just spin component measurement results which had the property of classical determinacy. Essentially Bell simply indexes these measurement results by states $\lambda \in \Lambda$ which are nothing more than lists of spin component values that would be found were certain measurements to be performed. Bell's $\lambda \in \Lambda$ are essentially the $\lambda_m \in \Lambda_m$ discussed previously in this Chapter.

The anti-realist option to be considered here might be called classically determinate anti-realism. According to this theory there are no unobservable particles in Bell-type experiments. According to this theory it does not even make sense to talk of measurement-independent spin component values. All one can talk about are measurement results. According to the theory, however, measurement results are classically determinate. This, of course, amounts to the introduction of states $\lambda_m \in \Lambda_m$--and we know, once again, that there is no density $\rho(\lambda_m)$ which can return the spin component correlation measurement statistics.

Classically determinate anti-realism can be contrasted with Bohr's anti-realism. In the Bohr theory, if we were to measure spin along $\vec{a}$ all we could talk about would be the spin along $\vec{a}$ measurement result and its statistics. We could not talk about spin along $\vec{b}$ since we were not at that time performing an experiment to determine a value for spin along $\vec{b}$. In the present anti-realist theory, however, each λ_m provides a value for the result of measuring spin along $\vec{a}$, but also provides values for the results of all the spin component experiments we did not perform but could have performed.

We can now see both why and when Bell's theorem is relevant to realists and anti-realists alike. The important kernel of the hidden variables assumption underlying the Bell result is the assumption of the classical determinacy of spin component values--be they measurement-independent values to be revealed by measurement or simply values that would be found were certain measurements to be performed.

Some realist theories make the hidden variables assumption, and so are ruled out on the assumption of Bell locality.[24] What has not generally been recognized is that there are versions of anti-realism which also make the requisite hidden variables assumption. Such versions of anti-realism will be ruled out on the assumption of Bell locality. To be sure, there are forms of anti-realism which do not make the hidden variables assumption (e.g., Bohr's anti-realism). What is absolutely crucial to those forms of anti-realism which are susceptible to Bell-type considerations is classical determinacy of measurement results.

This latter point may be illustrated (a little facetiously) by considering the thesis of Berkelean idealism which proceeds from the motto: 'to be is to be perceived'. In order to avoid problems with what happens to the tree in the quad when no one is around (or of unobserved parts of trees), the worthy bishop postulated that God perceives all things at all times, thus keeping the universe classically determinate and orderly. Thus when we measure spin along some direction $\vec{a}$ in the context of a Bell-type experiment and find a certain value, values for other spin components, qua results of measurements, will be similarly determinate (being perceived by God rather than us). Thus classical determinateness in an essentially idealistic theory implies violations of Bell locality--the mind-dependent spin component values can be indexed by states $\lambda \in \Lambda$.

It should be noted that Berkelean idealism is out of harmony with orthodox quantum mechanics on other grounds. An argument due to M.C. Robinson[25] suggests that if an unstable atom is measured and found to be undecayed, then by the projection postulate, immediately after measurement the state of the atom is the undecayed state. Since the unstable atom would be constantly watched by Berkeley's God, if it started out in the undecayed state, it would remain so for all time, contrary to the hypothesis that it was unstable. A watched quantum mechanical pot never boils.[26]

In discussing the implications of Bell's theorem for the realist/anti-realist debate, much has been made of the assumption of Bell locality. The usual view that quantum mechanics itself is Bell local will be subject to critical analysis in the next Chapter. In the remainder of this Chapter, in the context of a discussion of issues relating to the doctrine of epistemic realism, the concept of Bell locality itself will be subject to critical scrutiny.

SECTION FIVE: EPISTEMIC REALISM

Van Fraassen's[27] discussion of epistemic realism begins with a brief examination of the nominalist/realist debate in medieval philosophy. According to van Fraassen, the focus of that debate was the existence of causal properties used in explanations of regularities in nature. The nominalists, who denied the existence of causal properties and dispositions, were said by the realists not merely to view observed regularities as miraculous or inexplicable, but also to have no reason to expect observed regularities to continue. As van Fraassen puts it:

> The nominalist position in philosophy of nature would, in other words, lead to scepticism, to the impossibility of reasonable expectations about the future.[28]

Epistemic realism is the doctrine that reasonable expectations concerning future events are possible only on the basis of some understanding of, or reasonable certainty about, the causal mechanisms that produce those events.[29]

The core claim in van Fraassen's critique of epistemic realism is that epistemic realism, in the context of quantum spin (or polarization) correlation phenomena, (where such phenomena are viewed as regularities in nature), requires a common cause model for explanation. This, van Fraassen argues, leads to the demand for hidden variables, which he argues runs foul of Bell's theorem. But the argument is conditional, among other things, on Bell locality.

Consider two correlated events, such as outcomes of a Bell-type experiment with spacelike separated measuring devices. Call these events A and B respectively. A common cause for this correlation will, according to van Fraassen, have two properties: (a) relative to the common cause the two events will be independent, so if Z is the common cause:

1) $P(A \text{ and } B/Z=z) = P(A/Z=z)P(B/Z=z)$,

for all values z of Z; (b) (by Einstein locality) the common cause Z must lie in the intersection of the backward light cones of A and B. Van Fraassen asks:

> What would a causal theory of this phenomenon be like? It would postulate or exhibit a factor, associated with the source, that acts as a common cause for the two separate outcomes.[30]

In the context of a Bell-type experiment then, van Fraassen is operating with something like the following picture. The common cause of the correlated measurement events can be indexed by states $\lambda \in \Lambda$. We are to imagine particle-pairs to be emitted from the common particle source in classically determinate states λ; and the state λ of the pair at the source determines their responses (in a deterministic theory), or determines probabilities for responses (in a stochastic theory)[31] to orientations of the analyzers they later encounter when spacelike separated.

In a deterministic common cause model for the explanation of the spin correlation measurement statistics, relative to the $\lambda \in \Lambda$ at the source the correlation events are to be independent. We know from Bell's theorem that on the assumption of Bell locality there can be no density $\rho(\lambda)$ which will return the spin correlation measurement statistics. We have quantum mechanical predictions and some evidence (though not as unambiguous as van Fraassen supposes) that there are correlated events which cannot be embedded in a deterministic common cause model for explanation.[32] Van Fraassen concludes:

> Returning now to epistemology, let us again ask when it is possible to have reasonable expectations about future events. Assuming (as we surely all agree) that it is reasonable to base one's expectations on well-supported theories, we are reasonable to expect the persistence, wherever the relevant conditions obtain, of the correlations predicted by such theories. And this point is quite independent of whether we are provided with a causal explanation--or even the possibility thereof.[33]

This position is highly reminiscent of Bohr's anti-realism according to which quantum mechanics is to be assessed solely in virtue of the correctness of its statistical predictions.

Van Fraassen thinks he has shown, contrary to epistemic realism, that there is a case--the Bell case--where we have reasonable expectations concerning future events "not based on any understanding of . . . causal mechanisms." As noted by Stairs,[34] this argument of van Fraassen does not actually show that epistemic realism is false rather than that, lacking the possibility of a Bell local causal story for the correlation measurement statistics, the epistemic realist has no reason to expect the predicted correlations to persist into the future. That is, the epistemic realist could deny that he or she has reasonable expectations in the Bell case. Of course, as noted by Stairs, if the epistemic realist subscribes to the view that it is reasonable to base one's expectations on well-supported theories, then the conclusion against epistemic realism will go through.

There are three comments worth making at this juncture. First, there is no reason why the epistemic realist should accept the assumption that it is reasonable to base one's expectations on well-supported theories without severe qualification. The epistemic realist may claim to have reasonable expectations about the future only if a causal explanation or story can be provided. Now part of van Fraassen's point is surely this: that while it is true that to have a well-supported theory is very often to have a causal theory, this need not always be the case. In the case, however, where there is a well-supported theory and no possibility of a causal explanation of events the theory predicts, the epistemic realist might simply deny that we have reasonable expectations concerning those events. The epistemic realist might retort that in such a case, such expectations as we have are based on custom and habit rather than upon an understanding of mindindependent causal mechanisms.

So here is one sense in which van Fraassen's argument seems to beg the question against the epistemic realist. The epistemic realist may well buy Fine's dictum that quantum mechanics is a marvellous predictor but an incompetent explainer--and yet place no faith in the predictions until proper causal explanations of events predicted by quantum mechanics are forthcoming.

The second point to be made is this: even if the epistemic realist did accept van Fraassen's assumption that it is reasonable to base one's expectations on well-supported theories, the evidence at the time van Fraassen was writing (1980-1982) did not provide unambiguous support for quantum mechanics over Bell local realist theories.

This brings me to the third point. It is a requirement of epistemic realism that if there are regularly correlated events (e.g., spin correlation measurement statistics) then causal explanations must be forthcoming for those correlations. But it is not necessarily a requirement of epistemic realism that the causal story follow the local common cause model for causal explanation. One way to avoid the negative force of Bell's theorem for hidden variables strategies is to introduce action-at-a-distance or superluminary effects in the form of instantaneous transmission of signals between the analyzers in a Bell-type experiment. In a hidden variables theory with action-at-a-distance, the outcome of an experiment in one wing would not necessarily be independent of the direction along which spin is measured in the other wing of the apparatus. Neither, for that matter, would the probability distribution of the classically determinate states $\lambda \in \Lambda$[35] be necessarily independent of the two directions $\vec{a}$ and $\vec{b}$ along which spin is measured. Indeed, in such a case we might calculate the correlations as follows:

$$1)\quad E(\vec{a},\vec{b}) = \int_{\Lambda} A(\vec{a},\vec{b},\lambda)B(\vec{a},\vec{b},\lambda)\rho(\vec{a},\vec{b},\lambda)d\lambda)$$

Calculations in accord with (1) above entail no inequalities violated by quantum spin correlation statistics. The reason is clear: one has different probability densities ρ for different $\vec{a},\vec{b}$-experiments. There is no longer a common density $\rho(\lambda)$ from which all the measurement statistics are to be retrieved.

So one way to be an epistemic realist is the face of the Bell theorem is to opt for violations of Einstein locality. But doesn't this run counter to the special theory of relativity?

For the sake of argument, suppose that relativity theory does rule out violations of Einstein locality. In this case, the epistemic realist might argue that what is ruled out by the Bell result and the evidence which suggests it, is not the hidden variables assumption, as is commonly supposed, but rather the Einstein locality condition of relativity theory. The argument might run something like this: Einstein locality implies Bell locality, and since the assumption of hidden variables $\lambda \in \Lambda$ is incompatible with the assumption of Bell locality, and since the hidden variables assumption is needed for the epistemic realist's causal explanation of the spin correlation measurement statistics, then Bell locality must be abandoned and consequently Einstein locality must be abandoned. In this case the Bell result would be seen as ruling out the Einstein locality condition (on the epistemic realist's assumption of hidden variables). This is a drastic conclusion for Einstein locality is a deeply embedded presupposition of much work in modern physics.

In what follows I place the following constraint upon the enterprise of abandoning Bell locality in order to preserve the hidden variables assumption of states $\lambda \in \Lambda$ in the face of Bell's theorem: Bell locality must be abandoned in such a way that does not entail violations of Einstein locality. What is to be explored below is the possibility of having an Einstein local hidden variables theory of the Bell-type spin correlation measurement statistics. Such a theory could not be Bell local, and so a way must be found to block the implication of Bell locality by Einstein locality.

If Einstein locality can be separated from Bell locality, then a common cause model for causal explanation might still be found for the puzzling correlation statistics. In what follows it will be argued that a "super-deterministic" epistemic realist can have a common cause account of the spin correlation measurement statistics--and an account, moreover, that is Einstein local. This is important since, as previously noted,[36] it is Einstein locality and not Bell locality which is the locality condition of the special theory of relativity. It will be contended that van Fraassen begs the question against the super-deterministic epistemic realist. In the light of arguments to be presented a new interpretation of van Fraassen's argument against the epistemic realist will be offered. Finally, I will examine two suggestions from theoretical physicists which point in the direction of super-determinism.

Super-determinism will be explored below as one possibility. It is not claimed that super-determinism is true or has been shown to be true by the considerations below.

SECTION SIX: EINSTEIN LOCALITY AND BELL LOCALITY

In Chapter Three it was argued that Einstein locality implies Bell locality if the principle of system-apparatus independence is satisfied. It is important to examine how van Fraassen and other theorists have defended this principle of system-apparatus independence. One good way to do this is to consider what goes on in experiments to test Bell's theorem.

As noted in Chapter Three, the testing of Bell's theorem has proven to be a difficult matter whose stringent conditions have not as yet been satisfied. In the early (pre-Aspect) experiments, the analyzers were oriented well in advance of the running of the experiments. Such experiments, as tests of Bell's theorem, were deficient in that theoretically the setting of one analyzer could influence the state of affairs at the other analyzer, or could influence the distribution of $\lambda \in \Lambda$ at the source. All this without recourse to superluminary effects. It will be recalled that in a theory of the spin correlation measurement statistics with superluminary effects, we calculated the correlation statistics as follows:[37]

$$1)\quad E(\vec{a},\vec{b}) = \int_{\Lambda} A(\vec{a},\vec{b},\lambda)B(\vec{a},\vec{b},\lambda)\rho(\vec{a},\vec{b},\lambda)d\lambda$$

The situation which arises when the analyzers are in communication with each other (either locally or non-locally[38]) is reflected in the following probability equation:

$$2)\quad E(\vec{a},\vec{b}) = \int_{\Lambda} A(\vec{a},\vec{b},\lambda)B(\vec{a},\vec{b},\lambda)\rho(\lambda)d\lambda$$

Equation (2) reflects the view that the measurement result in the A-wing of a Bell-type experiment is not independent of the direction along which spin is measured in the B-wing, and vice-versa. That the state of affairs at the analyzers influences (either locally or non-locally) the state of affairs at the source, even though the analyzers themselves are not in communication, suggests the following probability equation:

$$3)\quad E(\vec{a},\vec{b}) = \int_{\Lambda} A(\vec{a},\lambda)B(\vec{b},\lambda)\rho(\vec{a},\vec{b},\lambda)d\lambda$$

Here, the distribution of the $\lambda \in \Lambda$ at the source is not independent of the orientations of the spacelike separated analyzers.

Probabilities calculated in accord with equations (1), (2) and (3) are not constrained by the Bell inequalities violated by the quantum spin correlation measurement statistics. The trouble is that the pre-Aspect experiments did not rule out use of either (1), (2) or (3) in favour of the Bell equation:

$$4) \quad E(\vec{a},\vec{b}) = \int_{\Lambda} A(\vec{a},\lambda)B(\vec{b},\lambda)\rho(\lambda)d\lambda$$

Bell's inequality constrains calculations in accord with (4). What is required is an experimental test of Bell's theorem which requires use of equation (4) as the probability equation and gives no grounds for supposing that we could use either (1), (2) or (3) above. The reason that the early tests of Bell's theorem were unsatisfactory is that in these experiments the analyzers were oriented well in advance of the running of the experiments. In this case, without introducing nonlocal effects in the form of superluminary action, it was at least theoretically possible to suppose that the correlations could be calculated using probability equations other than (4).

This is why the Aspect-type experiments have been considered to be so important. As D'Espagnat has noted:

> An experiment with instruments whose setting can be changed rapidly could exclude this possibility. The decision to measure a spin component with one detector would not be made until it was too late for any influence of that decision to reach the other instrument or the source, even at the speed of light, in time to alter the outcome of the second experiment.[39]

Aspect, using acousto-optical devices to give rapid adjustments of analyzer orientations, has performed experiments along these lines. As noted in Chapter Three, while the results of these experiments tended to provide some support for the predictions of quantum mechanics, it is not clear that the data yielded did not admit also of local realist models.

But what really matters here is not some minor point about the dubiety of the evidence. Even if such experiments were performed, with very swift analyzer changes, and the results were found to be in excellent agreement with quantum mechanics, they would *not* provide conclusive evidence against certain Einstein-local epistemic realist theories. It is open to the epistemic realist to say that the analyzers, whose settings have been changed fast enough to preclude analyzer-to-analyzer communication, as well as analyzer-to-source communication, nonetheless have their orientations pre-determined by causal factors in the past. As Clauser and Shimony put it, commenting on experiments with fast analyzer changes:

> However, even with such devices it is impossible to block the loophole completely. Since the backward light cones of the detection and adjustment events overlap, it may be claimed that events in the overlap region are responsible for determining the choices of the parameters a and b as well as the observed results. In this way the quantum mechanical coincidence counting rates can still be accounted for without any direct causal connection between opposite sides of the experiment, and hence without the introduction of action-at-a-distance.[40]

Factors may be postulated, that is, which determine that particle-pairs will be emitted in certain microstates $\lambda \in \Lambda$ if and only if certain $\vec{a},\vec{b}$-experiments are going to be performed--so that, as in equation (3), the distribution of λ is not independent of the experiment to be performed. In this case, the behaviour of the apparatus would not be independent of the states of the emitted systems, and so the principle of system-apparatus independence would not be satisfied. Under these conditions, Einstein locality would not imply Bell locality. Thus, even if we were confronted with an ideal Aspect-type experiment, we could, as epistemic realists, still insist that an Einstein local story could be told of the spin correlation measurement statistics. Such a story would not, of course, be Bell local. This position is the position of the super-deterministic epistemic realist.

According to super-deterministic epistemic realism, it is not only determined at the source how particles will respond to given analyzer orientations--it is also determined at the source (or at some time prior to particle-emission from the source) just what those orientations will be. Thus, particles are emitted in specific states $\lambda \in \Lambda$ just in case certain $\vec{a},\vec{b}$-experiments are going to be performed on them. Even though there is no communication between the analyzers, the distribution ρ of the hidden variables will be a function of $\vec{a}$ and $\vec{b}$ as well as λ. In such a case, the principle of system-apparatus independence is violated and probabilities are calculated using (3) rather than (4) above. As there are no Einstein non-local influences, we have here a local theory of the spin correlation measurement statistics in the sense of locality required by physics. Since we use probability equation (3), the theory will not be Bell local.[41]

In the FitzGerald-Lorentz hidden variables theory of Fresnel aether theory, factors were postulated (i.e., contractions of the apparatus) to account for the null result of the Michelson-Morley experiment. These factors were introduced in order to give an account of the behaviour of the apparatus so that, within the framework of aether theory, an account could be given of the experimental outcomes. In the theory of super-deterministic epistemic realism, factors are similarly introduced to explain the behaviour of the apparatus in Bell-type experiments. These factors enable us to produce a deterministic Einstein local account of the spin correlation measurement statistics.

Now the traditional view--van Fraassen attributes it to Kant[42]--states that the assertion of determinism has no empirical consequences. The idea is that any phenomena can be embedded into a deterministic story. Van Fraassen and others wish to show that this is not right. They claim that there are phenomena--quantum spin correlation phenomena--which cannot be embedded into a deterministic story. What super-deterministic epistemic realism does is to show that this is too strong a conclusion to reach--even on the basis of ideal Bell-type evidence. If one is a determinist even about the experiment being performed, then one can embed the quantum spin correlation phenomena into a deterministic, Einstein local, causal story--using a common cause model for causal explanation. The common cause simply causes more than van Fraassen supposes! The denial of the possibility that analyzer orientations are pre-determined begs the question against the case at hand.

Similar comments must be levelled against those theorists who appeal to the possibility of random fast analyzer changes in the context of Aspect-type experiments. Genuinely random analyzer changes would block the super-deterministic epistemic realist manoeuvre--but the invocation of such random phenomena would also beg the question against the super-deterministic epistemic realist through the invocation of indeterminism as regards analyzer behaviour.

Nevertheless, could there be evidence for random analyzer changes in the sense required to block the super-deterministic epistemic realist manoeuvre? Observation of the behaviour of the analyzers would not provide evidence that the principle of system-apparatus independence is satisfied. It is always possible that though the analyzers are changing their orientations in a seemingly random fashion, nevertheless these changes of orientation are pre-determined by factors in the past. This point may be illustrated as follows: consider a device consisting of a random number generator at point A connected to a digital readout at point B distant from A. Though the numbers on the readout change apparently at random, we would not say that the changes were independent of events taking place (in the past) within the random number generator. In Bell-type (or Aspect-type) experiments, it is possible that while the analyzers change their orientations apparently at random (like the numbers on the digital readout), the analyzer-change events are, even so, wholly dependent upon particle-emission events at the source (which are the counterparts of the events taking place within the random number generator). Thus, while the analyzers change their orientations 'at random', this apparently random behaviour is not independent of--and indeed is determined by--events at the source (or in the source's past).[43]

Indeed, it is possible that events at the source are genuinely random in the sense of being completely uncaused (like radioactive decay events), so long as the analyzer orientation events are not independent of these source events. What such a view requires is something like this: while the state λ of the emitted particle-pair may be assigned at random (by nature, say) at the source--genuinely at random --the subsequent $\vec{a},\vec{b}$-measurement is not independent (and indeed is determined by) this choice of λ. In a clear sense, the experiment performed would be as apparently random as the initial assignment of λ. In this case the probability density ρ would be a function of $\vec{a}$ and $\vec{b}$ as well as λ. We would use equation (3) to calculate the correlations. The account of the correlation statistics would be Einstein local though not Bell local (since the principle of system-apparatus independence is violated).

The indeterminist position just discussed is in some ways reminiscent of Lotze's indeterminism. As expounded by Jeans,[44] this is the view according to which all events lie on causal chains and that such chains, once started have no end in the future. Nevertheless, such chains may have capricious beginnings. In the indeterminist position discussed above, it is suggested that nature is capricious in assigning states λ to particle-pairs, but that subsequent events (e.g., analyzer orientations) are pre-determined by the albeit capricious events at the source.

Experiments in which analyzer-orientations apparently change at random do not provide evidence capable of ruling out certain brands of epistemic realism. If nature was capricious with the analyzer changes themselves (so that analyzer behaviour was independent of source behaviour), the experiments with such analyzer changes could provide evidence against the types of epistemic realism being discussed here. The trouble is that there is no evidence, and certainly none at present, which would allow us to distinguish such an account of analyzer behaviour from one in which we have random, but source-dependent, analyzer changes.

SECTION SEVEN: THE PRINCIPLE OF SYSTEM-APPARATUS INDEPENDENCE

In the literature on Bell's theorem the principle of system-apparatus independence is not standardly defended by appeals to random analyzer changes. On the contrary, the standard type of defence of the principle of system-apparatus independence makes essential reference to the concept of *experimenter freedom*. While this way of defending the principle of system-apparatus independence begs the question against the super-deterministic epistemic realist, it must nevertheless be given serious consideration.

Consider van Fraassen's comments on events in Bell-type experiments:

> In realistic examples, the events A and B are often outcomes of experiments. That the experiment is going to be done at all, is of course an independent point; what we are meant to explain causally is that the outcome is thus and so if the experiment is done. Hence the statement of the correlation takes the form: $P(A \text{ and } B/A^* \text{ and } B^*) = P(A/A^*)P(B/B^*)$. If there is spacelike separation between the two experiments, we suppose that either could be stopped at will before termination, and therefore that $P(A/A^* \text{ and } B^*) = P(A/A^*)$. This supposition may well be false, for it is conceivable that there is a pre-established harmony, and the experimenters are caused to perform the B^* experiment in just these cases in which the experiment A^* is performed and has outcome A. A little commonsense should help us here when we are discussing a specific, realizable experimental arrangement, though we must keep the pre-established harmony possibility in mind if we contemplate general conclusions.[45]

It is partly because van Fraassen *is* contemplating a general conclusion, to wit, that epistemic realism can be shown to be false by an appeal to the Bell-type evidence, that one must keep the possibility of pre-established harmony in mind.

In the passage quoted above, it is hinted that the pre-established harmony hypothesis goes against the grain of common sense. But what part of common sense does the pre-established harmony hypothesis run counter to? Common sense very likely has little to say concerning experiments designed to refute the possibility of certain types of causal explanation of quantum spin correlation phenomena. More importantly, van Fraassen hopes to show, contrary to the traditional view, that the assertion of determinism does have empirical consequences. In order to make this claim good, van Fraassen opts for indeterminism as regards experimenter behaviour. This begs the question against the super-deterministic epistemic realist in two ways: first, through the invocation of indeterminism; and secondly, through the invocation of mind-dependence. To wit, some events in a Bell-type experiment (e.g., selections of analyzer orientations) are the result of the experimenter having exercised his or her free will in making the choice.

The argument against epistemic realism due to van Fraassen depends crucially on the assumption of experimenter freedom. First Van Fraassen hints that we are free to perform or not perform the Belltype experiment. Secondly, and quite explicitly, he thinks we are free to stop the experiment in either wing after the initial emission of particles from the source. Thirdly, since the experimenters are not caused to choose certain orientations of the analyzers, they are presumably free to change the orientations of the analyzers in either wing of the experiment of their own volition. Thus it is that in the van Fraassen argument the behaviour of the apparatus is independent of the states $\lambda \in \Lambda$ of the emitted systems.

Van Fraassen is not the only one to have seen the importance of the principle of experimenter freedom in assessing the significance of Bell's theorem. Thus D'Espagnat comments on Bell's work:

> More generally, he . . . finally showed that some correlations between physical events taking place in different spacetime regions cannot be explained in terms of physical events in the overlap of the backward light cones of the two regions (unless the experimenter's freedom of choice is illusory, or some specific predictions of quantum mechanics are false; but these predictions have since been verified experimentally).[46]

One point which must be made immediately is this: if the allegations against the epistemic realist depend on invocations of experimenter freedom, then it is not clear that any of the experiments done to date (or even planned at this time) provide any evidence against the epistemic realist.

In order to exercise freedom of choice in the kind of way required to defend the principle of system-apparatus independence, experiments would be required with enormous analyzer separations so that particle flight times would be long enough to allow for the exercise of freedom of choice. Such experiments certainly have not been performed to date. Inasmuch as van Fraassen's case against epistemic realism depends on experimenter freedom then van Fraassen cannot claim any support from any of the experiments performed to date--their flight times are too short!

Stairs has also noted the importance of the principle of experimenter freedom to the Bell argument. Commenting on the view that experimenter freedom is illusory, he notes:

> It is, however, a disturbing prospect. If the experiments we choose to perform are not under our control, then one would expect this to lead to deep problems in making epistemological and methodological sense of science.[47]

While theorists make comments like this from time to time, it is not clear what force they are meant to have. Classical mechanics could be construed as a deterministic theory in precisely the sense objected to by Stairs. (One merely has to recall the predictive powers of the Laplacean demon to see this.) This point did not, apparently, cripple the classical mechanists when it came to making epistemological and methodological sense of science. Indeed, a cynic might suggest that troubles in epistemology and methodology are better associated with theorists who believe that there are regularly occurring correlations between events which cannot possibly be given a causal explanation!

Even Bell has invoked the principle of experimenter freedom of choice in order to justify the principle of system-apparatus independence.[48] The anticipated determinist response here is that invocations of experimenter freedom beg the question--at least against the super-deterministic epistemic realist. Clauser and Shimony represent this kind of attitude when they respond to Bell thus:

> Bell's reply . . . to this objection stresses the spontaneity of the experimenter's choice between $\vec{b}$ and $\vec{b}'$ and between $\vec{a}$ and $\vec{a}'$; but this answer seems to us to depend on too strong a commitment to indeterminism for his argument to be fully general.[49]

To be fair to Bell, he was recently asked the following question: can we believe in a deterministic universe in the light of the Bell-type experiments? Bell replied as follows:

> . . . one of the ways of understanding this business is to say that the world is super-deterministic. That not only is inanimate nature deterministic, but we, the experimenters who imagine we can choose to one experiment rather than another, are also deterministic. If so, the difficulty which this experimental result creates disappears.[50]

SECTION EIGHT: EXPERIMENTER FREEDOM AND THE QUESTION OF DETERMINISM

Can experimenter freedom be reconciled in some way with the kind of determinism needed in order to provide a causal account of the quantum spin correlation measurement statistics?

In terms of super-deterministic epistemic realism, we have seen that the apparent cost of a causal story for the Bell-type phenomena is experimenter freedom. On the other hand, if experimenters have freedom of choice, then no causal story is forthcoming of the Bell-type phenomena. (At least, no Einstein local causal story.)

On this way of putting things, the significance of the Bell result--and in particular its implications for Einstein local epistemic realism--depends on one's views concerning the free will/determinism debate. In what follows I will assess this debate with a view to clarifying some of its implications for the Bell result.

In the thirties, A.H. Compton attempted to deal with the question: is man a free agent? He commented:

> If the statements of the laws of physics were assumed correct, one would have to suppose . . . that the feeling of freedom is illusory, or if free choice were considered effective, that the statements of the laws of physics were . . . unreliable.[51]

Compton clearly saw some scope for conflict between the laws of physics and freedom of choice. Compton did not view the feeling of freedom to be illusory. He did, however, view the deterministic laws of classical mechanics to be unreliable. He even went so far as to look to the indeterministic quantum theory for a resolution of the conflict between physics and freedom.[52]

There is certainly something peculiar in the view that quantum mechanical indeterminacy somehow allows us to resolve the apparent conflict between the laws of physics and the feeling of freedom. Random behaviour is not free action, and neither is randomness a precondition for free action.

Perhaps, then, rather than dealing with the problem at hand by invoking stochastic physics, we should question the feeling of freedom itself. This was the line taken by Planck who commented:

> The principles of causality must be held to extend even to the highest achievements of the human soul. We must admit that the mind of each one of our great geniuses . . . was subject to the causal fiat and was an instrument in the hands of an almighty law which governs the world.[53]

Planck solves the problem by denying the reality of freedom.

What would, perhaps, be paradise regained would be a reconciliation of freedom with determinism--a reconciliation of Compton and his feeling of freedom with Planck and his almighty law. That there might be a reconciliation between freedom on the one hand, and determinism on the other, is clearly of great importance when one comes to assess the Bell theorem and its implications-- especially now that we see concepts such as 'super-determinism' and 'experimenter freedom' playing a vital role in arguments concerning the significance of Bell's theorem and the evidence which supports it. The position according to which freedom and determinism can be reconciled is called compatibilism. There are many versions of compatibilism. A recent and fruitful compatibilist programme is to be found in Davidson's doctrine of anomalous monism. This will now be presented and discussed.

Davidson's thesis of anomalous monism can be understood by recourse to the following three premisses:[54]

1) Psychological events, such as intentional actions, are caused by, and are the causes of, physical events.

2) When events are related as cause and effect there are laws ('a closed and deterministic system of laws . . .'), governing those events when appropriate described.

3) There are no precise psycho-physical laws.

Davidson comments on these premisses as follows:

> The three premisses, taken together, imply monism. For psychological events clearly cannot constitute a closed system; much happens that is not psychological, and affects the psychological. But if psychological events are causally connected to physical events, there must, by premiss two, be laws to cover them. By premiss three, the laws are not psycho-physical, so they must be purely physical laws. This means that psychological events are describable, taken one by one, in physical terms . . . Perhaps it will be agreed that this position deserves to be called anomalous monism: monism because it holds that psychological events are physical events; anomalous, because it insists that events do not fall under strict laws when described in psychological terms.[55]

The distinction between the physical and the psychological is made here on the basis of the types of event description used and not on the basis of ontological differences (e.g., as in cartesian dualism). Similarly, and contrary to Compton, the events involved in intentional action will be described as exhibiting intentionality and not as being random or as being parasitic on randomness. Finally, and contrary to Planck, psychological events, so described, will not be subject to an almighty causal law--though they may be when re-described in appropriate physical terms.

While it is not essential to my critique of van Fraassen's attack on epistemic realism that freedom and determinism be reconciled

--the feeling of freedom might after all be illusory and Planck might be right--it is nevertheless instructive to examine the consequences, for the interpretation of Bell's theorem and van Fraassen's arguments, of a successful reconciliation.

To this end, and for the sake of argument, suppose that Davidson's theory of anomalous monism is correct. How does anomalous monism help when we come to consider the significance of the Bell-type spin correlation measurement statistics? First, we know we can tell an Einstein local deterministic causal story about the events in a Bell-type experiment. The telling of this story requires that the behaviour of the apparatus (including the experimenter) be subject to a deterministic description whereby the actual setting of the apparatus, at the time of measurement of spin components on emitted particle-pairs, is not independent of the micro-state $\lambda \in \Lambda$ of the particle-pair being measured. The price paid in the telling of this story is that all events in the experiment, including those concerning the experimenter, must be described physically. Describing all the events physically--including selection-of-apparatus-orientation-events--allows us to have a deterministic causal explanation of the outcome events in a Bell-type experiment. We can thus satisfy the demands of Einstein local epistemic realism.

Secondly, by re-describing some of the events involved in a Bell-type experiment in psychological terms, we can allow for experimenter freedom in the sense of allowing the experimenter intentional and voluntary actions in the account we give (i.e., description) of analyzer-orientation selections. Describing some of the events, (the analyzer-orientation selection events), in a Bell-type experiment in psychological terms prevents us from bringing the outcome events of the experiment under Einstein local, deterministic causal laws.

Relative to a description of a Bell-type experiment in which analyzer orientations are the result of free experimenter choice, one is justified in claiming that the principle of system-apparatus independence is satisfied. In this case, violations of Bell locality entail violations of Einstein locality. Hence no Einstein local deterministic causal account will be forthcoming of the outcome events of a Bell-type experiment.

On the assumption that anomalous monism is correct, what we learn from a Bell-type experiment is this: (a) Einstein local deterministic causal explanations, as required by epistemic realism, are possible for the correlation statistics yielded by such experiments (even ideal experiments); (b) van Fraassen is correct that no Einstein local causal explanation of the correlation statistics is possible if experimenter behaviour is described as being free or intentional.

If anomalous monism is right then van Fraassen has not shown that epistemic realism is false, but rather that what is required by epistemic realism, (in this case, an Einstein local deterministic causal explanation of the outcome events in Bell-type experiments), cannot be satisfied under certain descriptions of the events in Belltype experiments. Under other (physical) descriptions of those same events the demands of epistemic realism can be met.

From the standpoint of anomalous monism, we make epistemological and methodological sense of science not by denying the determinism required for a causal account of the spin correlation measurement statistics in favour of experimenter freedom, but by allowing both determinism (under one type of event description) and experimenter freedom (under another type of event description). In another context Davidson has written:

> When the world impinges on a person, or he moves to modify his environment, the interactions can be recorded and codified in ways that have been refined by the social sciences and common sense. But what emerge are not the strict quantitative laws that we confidently expect in physics, but irreducibly statistical correlations that resist, and resist in principle, improvement without limit.[56]

To interpret these remarks of Davidson's in the present context it must be noted (as it has been by van Fraassen) that to bring the Bell-type correlation statistics under strict quantitative laws minimally requires a hidden variables interpretation or theory which postulates factors at the source which determine experimental outcomes at distant, spacelike separated analyzers. By describing experimenter events in psychological terms we find that the outcomes in a Bell-type experiment cannot be regarded as being so determined--and these outcome events end up as irreducibly statistical correlations. On the present view the mystery generated by the spin correlation measurement statistics lies not in the world, nor does it lie with probability theory. In order to generate a real mystery theorists have found it necessary to introduce into physics such intentional (non-physical scientific) notions as experimenter freedom of choice, without at the same time appreciating that the disastrous consequences for causal analysis were consequences of the psychological event descriptions rather than features of the real, mind-independent world.

The methodological lesson suggested by the doctrine of anomalous monism is that we should not mix social science with physical science. In the Bell case, both social science (with its psychological event descriptions) and physical science (with its purely physical event descriptions) have an interest in the behaviour of the experimenter. The former views such behaviour as intentional, the latter as deterministic.

If anomalous monism is correct then one might describe the 'van Fraassen fallacy' as being the faulty inference from the premiss that no causal analysis of the spin correlation measurement statistics is possible, given experimenter freedom, to the conclusion that no causal story can be told.

SECTION NINE: SUPER-DETERMINISM IN PHYSICS

Super-determinism, as it has been used in this Chapter, has the consequence that the experiments which we perform are pre-determined and subject to deterministic causal laws. Super-determinism is nothing more than Laplacean determinism, for as Laplace put it:

> We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at any given moment knew all the forces that animate nature and the mutual positions of the beings that compose it, if this intellect were vast enough to submit its data to analysis, could condense into a single formula the movement of the greatest bodies of the universe and that of the lightest atom: for such an intellect nothing could be uncertain; and the future just like the past would be present before its eyes.[57]

In particular, there would be no uncertainty about us, the experimenters. As Voltaire put it:

> It would be very singular that all nature, all planets, should obey eternal laws, and that there should be a little animal, five feet high, who, in contempt of these laws, could act as he pleased, solely according to his caprice.[58]

In this section I want to examine the way in which certain contemporary quantum theorists[59] have appealed to super-determinism as a way of treating some of the problematic and indeed paradoxical features of quantum mechanics.

a) Unified field theory: the work of Bohm and Hiley.

Einstein spent most of the latter part of his life trying to produce a unified field theory in which all the forces in nature could be derived from a common scheme or theory. As Suppes puts it:

> In the grand version of this scheme, for given boundary conditions the differential equations would have a unique solution for the entire universe, and all physical phenomena would be encompassed within the theory.[60]

Though Suppes himself is sceptical of the worth of the project of such grand system-building--preferring to view physics as essentially incomplete, it remains true that in recent years a growing number of physicists have become interested in unified field theories--theories which provide for a common understanding of the four known forces in nature: gravitational, electromagnetic, weak and strong forces.[61]

Bohm and Hiley[62] centre their analysis on the possibilities for a nonlinear field-theoretic approach to unified field theory of the kind in which Einstein was interested. Such an analysis would be deterministic.

Bohm and Hiley follow Einstein's suggestion that particles are field-theoretic entities. On such a view, a particle would be analyzed in terms of a field function which would be large only in a small spatial region. Einstein knew that non-linear equations admitted of such pulse-like solutions and it is the project of Bohm and Hiley to analyze quantum measurement statistics in terms of some such non-linear field theory.[63] They consider a Bell-type experiment concerning polarization components on photon-pairs resulting from positronium[64] decay. They comment:

> We are supposing that through non-linearity the movements of the positronium and of the 'particles' constituting the detectors, A and B, are coordinated in a certain way. The nature of the coordination is such that a positronium atoms decays into two photons with suitably related but well-defined polarizations, only when detectors are in a condition to absorb them . . .[65]

It is the hope of Bohm and Hiley that in a deterministic non-linear field theory the principle of system-apparatus independence may break down--even in Aspect-type experiments with fast switching times.[66] In this theory even the experimenter who assembles and operates the apparatus is to be treated field-theoretically.

Bohm and Hiley regard such a non-linear field theoretic approach as a hidden variables theory in which the probability density ρ is a function of the directions a and b along which polarization is measured, as well as of the hidden variables $\lambda \in \Lambda$. Since $\rho = \rho(\vec{a},\vec{b},\lambda)$ in this approach to the analysis of the correlation statistics, Bell locality will be violated. Since the principle of system-apparatus independence is not satisfied, violations of Bell locality need not constitute violations of Einstein locality. Bohm and Hiley add:

> In the model based on locally connected fields, this functional relationship arises because the observing apparatus and the observed system emerge together from the underlying field, in a way that requires certain restrictions on the distribution of hidden variables.[67]

Thus one specific proposal for an Einstein local deterministic account of the puzzling Bell-type correlation statistics involves the construction of a non-linear field theory in which the principle of system-apparatus independence breaks down.

b) The measurement problem: Schulman's theory.[68]

L.S. Schulman has proposed the adoption of super-determinism to avoid the so-called measurement problem in quantum mechanics.

Schulman considers the case where spin along $\overleftarrow{z}$ is measured on a silver atom using a Stern-Gerlach magnet. Using his notation, let $Ø_+$ and $Ø_-$ correspond to the values +1 and -1 respectively, for spin along $\overleftarrow{z}$. Let Ψ be a state of the measuring apparatus. Following Schulman, suppose that the initial state of the system-plus-apparatus is $Ø_+\Psi$; where **H** is the total Hamiltonian and t is the time the system takes to pass through the apparatus, the final state will be:

1) $Ø_+'\Psi_+' = U(Ø_+\Psi)$, $U = \exp(i\mathbf{H}t)$.

Here $Ø_+'$ is the spin +1 along $\overleftarrow{z}$ state and the prime denotes a change of coordinates. The state Ψ_+' is the final state of the apparatus with the pointer showing spin +1 along $\overleftarrow{z}$. In a similar way we understand the equation:

2) $Ø_-'\Psi_-' = U(Ø_-\Psi)$.

Schulman suggests that we consider an atom polarized initially along the $\overleftarrow{x}$-axis. The state of the system relative to spin along $\overleftarrow{z}$ is:

3) $(Ø_+ + Ø_-)/\sqrt{2}$

By a dynamical analysis of the measuring process using the Schroedinger equation, measurement has the following consequence:

$$4)\quad 1/\sqrt{2}\phi_+'\Psi_+' + 1/\sqrt{2}\phi_-'\Psi_-'$$
$$= U[1/\sqrt{2}\,(\phi_+ + \phi_-)\Psi]$$

The measurement problem is this: how do we get from this state (4) to an eigenstate of spin along $\overleftarrow{z}$. The usual answer makes use of the projection postulate: if the measured value of spin along $\overleftarrow{z}$ is +1 (say) then the state of the system immediately after measurement is the eigenstate of spin along $\overleftarrow{z}$ corresponding to that eigenvalue (i.e., ϕ_+).

It is Schulman's contention that equations (1)-(4) above are true for the majority of apparatus states Ψ. But he claims that there are special states Ψ such that:

$$5)\quad U[1/\sqrt{2}\,(\phi_+ + \phi_-)\Psi] = \phi_+'\Psi_+' \text{ or } \phi_-'\Psi_-'$$

The proposed states Ψ are to be macroscopically indistinguishable from ordinary states of the apparatus. These states Ψ are to have many 'precise coherences' that manage to produce equation (5) where a definite result of measurement for spin along $\overleftarrow{z}$ emerges. Schulman hypothesizes:

> In all actual experiments the initial state is special; the macroscopic state of the apparatus is perfectly attuned to giving a definite result for the particular input wave function of the system to be measured.[69]

Schulman's theory is super-deterministic and is such that the principle of system-apparatus independence is violated. As Schulman puts it:

> I require the entire universe be one tightly, coherently interconnected system, a single wave function. In particular each apparatus arranges itself into rare states so as to provide definite output for the atom coming its way. There can be no deviation from the plan and all time evolution is deterministic evolution under the Schroedinger equation . . . The subjective perception of being able to control an experiment or change it at will must be considered in the present theory to be illusory.[70]

In this theory the initial state of the measured system (i.e., $(\phi_+ + \phi_-)/\sqrt{2}$) becomes ϕ_+ (say) at the end of the experiment as the result of appropriate interactions with the microscopic components of the apparatus, which, according to Schulman, are poised to give just the right "pushes and pulls". By considering special apparatus states Ψ, Schulman hopes to avoid the traditional arguments for the impossibility of getting definite measurement results using just the Schroedinger apparatus.[71]

The case studies of Bohm and Hiley and of Schulman show merely that certain speculations in the field of theoretical physics have tried to exploit super-determinism in the sense relevant to the discussion of epistemic realism in this Chapter. The super-determinist rejoinder to van Fraassen's critique of epistemic realism does not, of course, depend on the specific programmes of either Bohm and Hiley or of Schulman being carried to completion. The case studies are important from a methodological point of view: in both cases theorists have shown a willingness to abandon the principle of experimenter freedom in order to gain a more satisfactory account of physics (at least from a conceptual point of view).

It has been suggested here that super-determinism is one possibility which might be exploited in the face of Bell's Theorem. The arguments above do not show super-determinism to be true. Rather, they explore the consequences for Bell's Theorem of the assumption that super-determinism is true.

FOOTNOTES

[1] Putnam (1978), p. 18.

[2] Clauser and Shimony (1978), D'Espagnat (1978), Van Fraassen (1980), (1982).

[3] For example, Bohrian anti-realism.

[4] That is, theories threatened by Bell's theorem.

[5] Super-deterministic physics will be discussed later in this Chapter. What matters here is that in such physics, the assumption of system-apparatus independence can break down.

[6] In the first stage of the argument it will be argued that there may be classically indeterminate realist theories. Such theories would not violate Bell locality. In the second stage of the argument it will be seen that certain realist theories may be Einstein local though not Bell local.

[7] Devitt (1984), p. 104.

[8] Fine (1986), p. 94.

[9] Einstein. Quoted by Fine (1986), p. 95.

[10] Eg., London and Bauer (1939). See also Wigner (1973), pp. 380-382.

[11] Cartwright (1983), pp. 4-5.

[12] MacIntosh (1984), p. 522.

[13] I am deliberately remaining neutral here on the Ival/Mval-sentence distinction.

[14] Fine (1974), p. 261.

[15] As noted in Chapter Two, there is a wide variety of hidden variables strategies so one need not necessarily proceed in this way.

[16] I do not know if anyone holds either position.

[17] Ival and Mval-sentences are defined at the beginning of this section.

[18] We could think of pairs of particles with simultaneous exact values for position and momentum but with no measurement-independent values for spin components.

[19] Again, I do not know if anyone holds either position.

[20] The A-wing and the B-wing refer to parts of the apparatus in the Bell-type experiment discussed in Chapter Three.

[21] I do not know if this is the only way to proceed. It is at least a possibility. What matters for the argument is that there is at least some way to explain the measurement result $S_1(\vec{a}) = +1$.

[22] Vaihinger. Quoted in Edwards (ed.), vol. 7-8, (1967), p. 222.

[23] Price (1940), p. 141.

[24] As to whether there can be realist options which do not make the requisite hidden variables assumption depends on one's view of the realist options discussed in Section Three. These options are admittedly difficult to make good sense of.

[25] Robinson, (1969).

[26] I assume, of course, that Schroedinger-type evolution only takes place when systems are not being measured or watched.

[27] van Fraassen (1982).

[28] Ibid., p. 25.

[29] As it stands this is not really an adequate definition of epistemic realism, even though this is the way van Fraassen puts it. To be fully realist, the causal mechanisms must be mind-independent.

[30] van Fraassen (1982), p. 31.

[31] See Chapter Three for a discussion of the difference between deterministic and stochastic hidden variables theories.

[32] The reason for mentioning deterministic common cause models is this: later in this Chapter it will be argued that there are deterministic common cause models for the causal explanations of the quantum spin correlation measurement statistics. Such models will be Einstein local though not Bell local.

[33] van Fraassen (1982), p. 36.

[34] Stairs (1984).

[35] In this and subsequent references to states $\lambda \in \Lambda$ it is assumed that the λ are assigned to systems at the source. So we refer here to a distribution of λ at the source. Since we are dealing with the case of the deterministic epistemic realist, the λ at the source will determine responses to analyzers spacelike separated from the source.

[36] See the distinction between Einstein locality and Bell locality in Chapter three.

[37] This equation could be used in the case where the analyzers and the source were each in local communication with each other. This would be possible if the analyzers were oriented well in advance of the running of the experiment. Again, the $\lambda \in \Lambda$ are assigned to systems at the source.

[38] By 'locality' I mean Einstein locality.

[39] D'Espagnat (1979), p. 178.

[40] Clauser and Shimony (1978), p. 1921.

[41] Equation (3) violates the requirement of Bell locality that the density ρ be a function of λ alone.

[42] van Fraassen (1982), p. 33.

[43] Hence the talk of 'seemingly random' behaviour.

[44] Jeans (1945), p. 210.

[45] van Fraassen (1982), p. 28 (my italics).

[46] D'Espagnot (1981), p. 205 (my italics).

[47] Stairs (1984), p. 354.

[48] Clauser and Shimony (1978), p. 1900.

[49] Ibid., p. 1900.

[50] Bell (1986), p. 47. My work does not depend on Bell's analysis.

[51] Compton (1940), p. xi.

[52] A similar kind of claim can be found in Jeans (1945).

[53] Planck. Quoted Jeans (1945), p. 213.

[54] Davidson (1976), p. 102.

[55] Ibid., pp. 102-3.

[56] Ibid., p. 102.

[57] Laplace. Quoted Kline (1953), p. 292.

[58] Voltaire. Quoted Kline (1953), p. 293.

[59] Bohm and Hiley (1981); Schulman (1986).

[60] Suppes (1984), p. 111.

[61] See Crease and Mann (1986) for a readable historical account of unified field theory.

[62] Bohm and Hiley (1981).

[63] Bohm and Hiley provide only a qualitative analysis. Precise details in terms of definite calculations are not given.

[64] Positronium is an unstable corpuscular configuration resembling an atom of hydrogen, but consisting of a positron (instead of a proton) and an electron. It decays by annihilation into two or three photons.

[65] Bohm and Hiley (1981), p. 539.

[66] Ibid., p. 540.

[67] Ibid., p. 544.

[68] Schulman's approach to the measurement problem was brought to my attention by Professor Michael Revzen, visitor to the Department of Physics at the University of Alberta.

[69] Schulman (1986), p. 692.

[70] Ibid., p. 693.

[71] I have not examined this issue at all. Schulman's paper is largely programmatic. I have no idea at the time of writing what the final verdict will be as regards this super-deterministic proposal.

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CHAPTER SIX

QUANTUM MECHANICS AND BELL'S THEOREM

> Whereas the conventional theory deals with ensembles of quantum systems that have been 'preselected' on the basis of some initial observation, we shall deduce from it probability expressions that refer to ensembles that have been selected from combinations of data favouring neither past nor future. A theory that concerns itself with such symmetrically selected ensembles . . . will contain only time-symmetric expressions for the probabilities of observations.
>
> Aharonov, Bergmann and Lebowitz[1]

SECTION ONE: INTRODUCTION

Bell derives a contradiction from two assumptions: that there are hidden variables and that quantum phenomena are Bell local. Since no interpretation of quantum mechanics, which predicts the puzzling quantum spin correlation measurement statistics, can satisfy both assumptions, then at least one of the assumptions must be abandoned in the light of Bell's theorem. This is Bell's dilemma. Since it is now fairly common for theorists to refer to Bell's theorem as a 'no hidden variables' proof, it is clear that there has been a strong tendency to resolve Bell's dilemma in favour of keeping the assumption of Bell locality--and so abandoning the hidden variables assumption.[2]

One possibility not standardly discussed is that under certain circumstances quantum mechanics may actually require hidden variables (i.e., classically determinate states) $\lambda \in \Lambda$ to determine measurement results. If quantum mechanics does require such states, then it would (by Bell's theorem) seem to imply its own Bell non-locality.

The third question with which this study is concerned is this: is the standard view that quantum mechanics has no commitment to hidden variables correct? This question will be examined by considering a probability rule due to Aharonov, Bergmann and Lebowitz,[3] and its consequences for the interpretation of quantum mechanics. This rule will be referred to as the ABL-rule. Only recently have physicists begun to explore the consequences of the ABL-rule for the interpretation of quantum mechanics.[4] This chapter contributes to this exploratory exercise by considering three applications of the ABL-rule: (1) the measurement problem as manifested in the paradox of Schroedinger's cat; (2) The Kochen and Specker 'no hidden variables' proof, and (3) the Bell argument.

In discussing the ABL-rule some cautions must be borne in mind. First, the ABL-rule is not some new dynamical rule which allows us to analyze how systems behave over time. Rather, it is a phenomenological probability rule which relates certain actual measurement results to certain probabilities for other measurement results which we could have performed. Secondly, the ABL-rule is not derived here from first principles --- though considerations are put forward to suggest that the ABL-rule can be derived from standard quantum mechanics. Thirdly, and related to this last point, it is fairly clear that a number of physicists do regard the ABL-rule as part of quantum mechanics. This chapter, then, is an exploration of the consequences for quantum mechanics of the ABL-rule. In view of these consequences, it may well be the case that one would wish to block the inclusion of the ABL-rule from standard quantum mechanics.

Before going on to present and discuss the ABL-rule, some preliminaries are in order. Einstein's *reality criterion*[5] holds that if it is certain (i.e., with probability one) that without disturbing a system, measurement of observable A will yield eigenvalue a_i, then there is an element of reality corresponding to this fact--the observable A takes the value a_i independently of measurement.

That observables take values independently of measurement and that measurement can reveal those measurement-independent values are strong assumptions.[6] The Bell argument itself does not require that either or both of these assumptions be made.

The hidden variables $\lambda \in \Lambda$ required by the Bell argument need not be lists of pre-possessed values for spin components capable of being revealed by measurements. On the contrary, it suffices if what is classically determinate is measurement results--that is, it suffices if the $\lambda \in \Lambda$ are just lists of values that would be found were measurements of spin components to be performed. From the mere fact that a spin component measurement result is determinate in this sense, it does not follow that the value for the spin component revealed upon measurement was in fact a pre-existing or measurement-independent property of the measured system.

When it was said above that quantum mechanics may actually require hidden variables under certain circumstances, it was implied only that under those circumstances quantum mechanics may define a set of measurement probabilities such that measurement results for a certain class of observables are classically determinate.

SECTION TWO: TIME-SYMMETRY AND THE QUANTUM MEASUREMENT PROCESS

As far back as 1964, Aharonov, Bergmann and Lebowitz argued that the basic laws of quantum mechanics are time-symmetric. Their arguments were concerned to combat the view that asymmetry in the direction of time--even thermodynamic anisotropy--enters quantum mechanics through the theory of measurement.

After all, one might reason that measurement changes the quantum state of a system discontinuously (according to the projection postulate) in a way not given by the (reversible) Schroedinger equation. Is this feature of quantum mechanics a ground for asserting that the basic laws of quantum mechanics are time-asymmetric? Aharonov et. al. do not believe so. They note that aside from considerations of entropy, quantum mechanics is concerned almost exclusively with the prediction of probabilities of outcomes of future measurements on the basis of the results of earlier observations. However, they claim:

> . . . the customary assignment of a state vector to a system on the basis of the most recent preceding observation may be somewhat arbitrary. This assignment is based on the intuitive notion that the measurement is the 'cause' and the quantum state is the 'effect', and that cause must precede effect in time. Also, perhaps, there is the notion that the quantum state of a system embodies the maximum information available to us about the system at any time; ordinarily, we can know the outcome of all observations in the past but not of those yet in the future.[7]

Aharonov et. al. believe there may be circumstances under which this is not the only way to view quantum measurements.

Aharonov et. al. consider ensembles of quantum systems that have been 'pre-selected' on the basis of some initial observation and 'post-selected' on the basis of some final observation. The resultant probability expressions will refer to ensembles of systems in the interval between two measurements. The probability expressions will be time-symmetric, favouring neither the initial nor the final measurement.

In searching for such probability expressions, Aharonov et. al. consider systems which are subjected to sequences of measurements, each of which is complete (in the sense of revealing a unique quantum state of the system). Their analysis begins with the orthodox assumption that initially each system in an ensemble has yielded a specified non-degenerate eigenvalue of some observable A. Furthermore, it is assumed that the observables under consideration are constants of the motion. From the point of view of this study this is an important assumption. It is intended in what follows to use the arguments of Aharonov et. al., which concern systems over time, to make some points about the quantum mechanical hidden variables issue, which traditionally concerns systems at some specific time.[8] Hence the interest in observables whose probabilities for measurement outcomes are constant over time (in the period of interest). Furthermore, the actual examples cited by those who argue that it is impossible to provide a hidden variables interpretation of quantum mechanics typically involve consideration of observables which are constants of the motion: this is as true of Kochen and Specker as it is of Bell.

Consider three observables which are constants of the motion: A, X and B. Consider an ensemble of systems all of which have yielded the eigenvalue a of observable A at time t_1. The probability, in an ensemble of systems which have yielded eigenvalue a for A that the next following measurement of X will yield eigenvalue x is:

1) $P(x/a) = Tr(A_a X_x) = |(\emptyset_a, \emptyset_x)|^2$,

where A_a denotes the idempotent operator $|a\rangle\langle a|$, and $\emptyset_a$ is the eigenstate of A corresponding to the eigenvalue a, etc. As shown by Aharonov et al., the probability in an ensemble of systems which have yielded at time t_2 the eigenvalue b of observable B, that an immediately preceding measurement of X would have yielded eigenvalue x is to be analyzed in a similar manner:

2) $P(x/b) = |(\emptyset_b, \emptyset_x)|^2$.

Equation (1) is the prediction formula and equation (2) is the retrodiction formula. Conditional quantum mechanical measurement probabilities are thus treated as being time-symmetric.

Consider now an ensemble of systems which have yielded eigenvalue a for A at t_1 and eigenvalue b for B at t_2. What is the probability that measurement of X at t_n, $t_1<t_n<t_2$, would have yielded the value x? According to Aharonov et al., this probability is to be understood classically as:

3) $P(x \text{ at } t_n/a \text{ at } t_1 \text{ and } b \text{ at } t_2)$

$$= \frac{P(x \text{ at } t_n \text{ and } b \text{ at } t_2/a \text{ at } t_1)}{P(b \text{ at } t_2/a \text{ at } t_1)},$$

which (suppressing references to time indices) may be rendered in quantum mechanics as:

$$4)\quad P(x/a \text{ and } b) = \frac{1}{H(a,b)}\,\mathrm{Tr}\,(A_a X_x B_b X_x),$$

where

$$5)\quad H(a,b) = \Sigma_{x'}\,\mathrm{Tr}(A_a X_{x'} B_b X_{x'}),$$

where the set [x'] is the set of eigenvalues of X. Equation (4) may be put into Hilbert space language as follows:

$$6)\quad P(x/a \text{ and } b) = \frac{|(\emptyset_x,\emptyset_a)|^2\,|(\emptyset_b,\emptyset_x)|^2}{\Sigma_{x'}\,|(\emptyset_{x'},\emptyset_a)|^2\,|(\emptyset_b,\emptyset_{x'})|^2}$$

Equation (6) will be referred to as the ABL-rule. It is easy to see that equation (6) entails that in the interval $t_1 < t_n < t_2$, $P(A=a) = P(B=b) = 1$, regardless of whether A and B commute. The probabilities referred to in (6) refer to an ensemble of systems which have been selected on the basis of specified initial and final observations. Aharonov et al. comment:

> . . . in experimental physics selections are frequently based on combinations of initial and final characteristics. Consider a beam of particles that enters a cloud chamber or similar device controlled by a master pulse. For the device to select an event as belonging to a sample to be evaluated statistically, the particle must enter the chamber and, prior to the onset of any manipulation by magnetic fields etc., satisfy certain requirements. But in order to be counted the particle must also activate the circuits of counters placed below the chamber; thus we make the selection on the basis of both the initial and the final state . . . Thus our formal treatment of initial and final states as an equivalent footing is not inconsistent with experimental procedures used in some investigations.[9]

Another example of the experimental significance of pre-selection and post-selection comes from a consideration of Stern-Gerlach experiments. Here we imagine a beam of silver atoms prepared initially in an eigenstate of spin along $\overleftarrow{z}$ (say the eigenstate that corresponds to spin +1 along $\overleftarrow{z}$). We can imagine these atoms to be then subjected to a measurement of spin along $\overleftarrow{y}$. For various purposes we could well be interested in those systems which initially yielded the value +1 for spin along $\overleftarrow{z}$ and subsequently yielded the value -1 for spin along $\overleftarrow{y}$. Indeed, in quantum mechanical investigations generally, we standardly consider cases where systems are initially prepared in an eigenstate of some observable A and are then subjected to measurements to determine values for another observable B. Suppose we are interested in those systems which yielded the value b_j for B. Those systems are naturally described as belonging to the ensemble of quantum systems which yielded value a for observable A at t_1 (the time of preparation) and subsequently yielded at t_2 (the time of measurement) the value of b_j for B. The ABL-rule allows us to discuss some of the statistical properties of this ensemble of quantum systems selected on the basis of initial and final states.

It will be argued below that the systems appealed to in the 'no hidden variables' proofs are amenable to analysis in terms of the ABL-rule. It must be noted immediately that the probabilities referred to in the ABL-rule are *ex post facto* probabilities or counterfactual probabilities: that is, probabilities that measurement *would have* revealed eigenvalue x of X at t_n given A = a at t_1 and B = b at t_2, and where $t_1<t_n<t_2$.

Since the ABL-rule requires that $P(A=a)=P(B=b)=1$ in the interval $t_1<t_n<t_2$ of interest, regardless of whether A and B commute, it appears that the strict (orthodox) connection between measurement outcomes and eigenstates cannot be maintained in this special case. According to the orthodox view, measurement of A is certain to yield eigenvalue a if and only if the state $\emptyset$ of the system is the eigenstate of A corresponding to the eigenvalue a. In traditional hidden variables strategies, the attempt is made to break this connection between eigenstates and eigenvalues so even though the probability that $A=a$, relative to quantum state $\emptyset$, is less than unity, nevertheless a system in that state may be such that $A=a$.

In the case of the ABL-rule, something even stronger may be claimed: that quantum mechanics itself determines that some observables would yield certain results upon measurement with probability one, even when the state of the system is not an eigenstate of those observables. In the present case, assume that the initial state of a system is an eigenstate of A and that the observable B does not commute with A. Suppose that $A=a$ at t_1 and that $B=b$ at t_2, $t_1<t_n<t_2$. In this interval, $P(B=b)=1$ regardless of the fact that B does not commute with A. Indeed, $P(A=a)=1$ during the interval as well. The ABL-rule thus allows us, apparently on quantum mechanical grounds, to extend the certainty as regards measurement outcomes that we normally associate with eigenstates of those observables to other states of those observables during certain intervals of interest. These remarks must be qualified.

The first thing to be emphasized is that the ABL-ensembles are not the same as the usual (unqualified) ensembles which are standardly discussed in connection with quantum statistical issues. The ABL-ensembles turn out to be sub-ensembles of the usual quantum ensembles. This is because the familiar quantum ensembles are ensembles of systems which have yielded some value a for observable A at some time t. The ABL-ensembles are ensembles of systems which have yielded A=a at t_1 and B=b at t_2. These ensembles are thus qualified on the basis of an initial and final selection.

If P(A=a)=1 then we say that the A-measurement result is determinate: measurement of A is certain to yield eigenvalue a. The claim above, then, is this: there are certain well-defined sub-ensembles of the usual quantum mechanical ensembles, in which measurement results for certain non-commuting observables are determinate. In the interval $t_1<t_n<t_2$, P(A=a)=P(B=b)=1, so in that interval the A-measurement result and the B-measurement result are simultaneously determinate regardless of whether A and B commute.

As will be seen in the case studies that will be discussed shortly, the ABL-ensembles turn out to be very important as regards the analysis of the so-called 'no hidden variables' proofs.

It has been claimed above that in the ABL-ensembles the orthodox link between eigenstates and eigenvalues is broken for certain observables. Aharonov et.al. comment:

> From a purely operational point of view, one might eschew the assignment of quantum states altogether and instead rely entirely on probability statements referring to carefully defined ensembles. However, as long as one does assign quantum states to physical systems, it appears defensible to do so either in reliance on the complete observation immediately preceding (as is customary) or on the next following, depending on the circumstances. This ambiguity indicates that the quantum state of a system, though undoubtedly containing some elements of 'reality' independent of any observer, also has subjective elements.[10]

That $P(A=a)=P(B=b)=1$ in the interval of interest highlights the importance of dealing with observables which are constants of the motion. As long as we consider such observables and the interval between measurements, we can talk meaningfully about probabilities for outcomes we would have found had we performed appropriate experiments. The ABL-rule furnishes the means to talk about those outcomes and their probabilities. If the observables we were dealing with were not constants of the motion, then there would be no straightforward inference from measurement results at t_1 and t_2 to probabilities for measurement results in the interval $t_1<t_n<t_2$.

SECTION THREE: THE PARADOX OF SCHROEDINGER'S CAT

The difference between orthodox quantum mechanics and quantum mechanics incorporating the ABL-rule can be seen by considering the paradox of Schroedinger's cat, a piece of burlesque presented originally in 1935 by Schroedinger in order to illustrate some of the absurdities latent in the orthodox interpretation of quantum mechanics. The version of the cat paradox presented here is not Schroedinger's version but is a version tailored to suit the assumptions underlying the ABL-rule.[11]

The design of the Schroedinger apparatus will be assumed to be as follows: there is a particle-source which prepares spin-1/2 particles in the eigenstate corresponding to the eigenvalue spin +1 along $\vec{a}$. The particle prepared in this spin state is imagined to be fired into the Schroedinger cat box. On entering the cat box the particle encounters a Stern-Gerlach apparatus oriented to measure the observable spin along $\vec{b}$ ($\vec{a} \neq \vec{b}$). If the particle shows spin +1 for spin along $\vec{b}$ then a circuit will close and the cat (also inside the box) will be electrocuted.

By quantum mechanics without the ABL-rule, a particle in the eigenstate $\emptyset$ of spin along $\vec{a}$, corresponding to the eigenvalue +1, will be such that:

1) $\emptyset = c_1\Psi_1 + c_2\Psi_2$,

for arbitrary c_1, c_2, and where Ψ_1 and Ψ_2 are eigenstates of spin along $\vec{b}$ corresponding respectively to the eigenvalues +1 and -1. On the orthodox view, a particle in state $\emptyset$ does not take an eigenvalue for spin along $\vec{b}$. The story continues that when a measurement of spin along $\vec{b}$ is performed, $\emptyset$ will reduce (stochastically) to either Ψ_1 or Ψ_2 (depending on the value revealed). On the orthodox view the measurement of spin along $\vec{b}$ takes place when the experimenter opens the box and finds either a dead cat (corresponding to spin +1 along $\vec{b}$) or a live cat (corresponding to spin -1 along $\vec{b}$). Say the experimenter finds a dead cat upon opening the box. Immediately after measurement the state of the cat is the dead cat eigenstate. In that state it is certain that the cat is dead.

On the orthodox view, however, immediately prior to measurement the state of the cat is a linear superposition of cat-vitality observable eigenstates. At such a time the probability that we would find a live cat upon measurement is 0.5 (and similarly there is a probability of 0.5 that we would find a dead cat upon measurement). As noted at several points during this study these orthodox probabilities *do not* represent mere ignorance concerning the true state of the cat. Prior to measurement, the cat-vitality observable measurement result is simply not determinate.

In terms of quantum mechanics with the ABL-rule, we get a very different picture of the cat problem. Here we have an initial measurement corresponding to preparation of the particle into the eigenstate corresponding to spin +1 along $\vec{a}$. Let this take place at t_1. Let our subsequent observation of the cat take place at t_2 ($t_1<t_n<t_2$). Suppose at t_2 that we find a dead cat upon measurement. This cat-vitality observable measurement result corresponds to the particle fired into the box taking the value +1 for spin along $\vec{b}$. The ABL-rule implies that in the interval $t_1<t_n<t_2$:

2) $P(S(\vec{a}) = +1) = P(S(\vec{b}) = +1) = 1$

That is, the spin along $\vec{b}$ measurement result--the value for spin along $\vec{b}$ were a spin along $\vec{b}$ measurement to be performed--is at no time undetermined or indeterminate in the interval of interest. Consequently, the result of measuring the cat-vitality observable is at no time undetermined or indeterminate either.[12]

Thus, according to orthodox quantum mechanics, though we found a dead cat upon measurement (at time t_2), the result could have been different. At the time of measurement there was a probability of 0.5 that we could have found a live cat (rather than the dead cat we actually found). Furthermore, the result of measurement at t_2 implies nothing for the result of measurement of the cat vitality observable which we could have performed at $t_2 - \Delta t$. So though we found a dead cat at t_2, had we measured the cat vitality observable at $t_2 - \Delta t$ we could have found a live cat. Indeed, this outcome has a probability of 0.5.

In terms of quantum mechanics with the ABL-rule, if we find a dead cat at t_2 then the result was determinate at $t_2 - \Delta t$. So had we measured the cat vitality observable at $t_2 - \Delta t$ we would have found a dead cat. What is unknown until we perform the measurement at t_2 is the value we would find were we to perform the measurement. Performing the measurement at t_2 removes our ignorance and we then know, on the basis of the result of measuring the cat vitality observable at t_2, what the value would have been had we performed the measurement at $t_2 - \Delta t$.

SECTION FOUR: FREEDOM AND CAUSALITY

For the arguments to come, the following principles will be of interest.

1) The Principle of Experimenter Freedom: according to this principle, we are free to measure any observable we choose.

Thus, rather than measure the observable B at t_2 we could have measured C instead. This principle is essentially no different from the principle of experimenter freedom discussed in Chapter Five in connection with Bell's theorem. Conditional upon certain compatibilist doctrines being acceptable, it was argued in the last Chapter that this principle need not conflict with certain determinist perspectives relevant to the analysis of various locality conditions.

2) The Principle of No Retroactive Causality: according to this principle, measurement of an observable B at t_2 does not bring it about at t_2 that the B-measurement result is determinate in the interval $t_1<t_n<t_2$ to which we are applying the ABL-rule.

In other words, measuring B at t_2 does not change the past. Thus we do not want to say, with regard to the cat paradox above, that the spin along $\overleftarrow{b}$ measurement result is determinate in the interval of interest in virtue of spin along $\overleftarrow{b}$ being measured at t_2--so that had we not measured spin along $\overleftarrow{b}$ but had measured, say, spin along $\overleftarrow{c}$, the measurement result for spin along $\overleftarrow{b}$ would be undetermined or indeterminate.

Principles (1) and (2) will be of great importance when the ABL-rule is applied to the situation underlying Bell's theorem later in this Chapter.

SECTION FIVE: THE KOCHEN AND SPECKER ARGUMENT

The Kochen and Specker argument according to which hidden variables cannot be introduced into quantum mechanics, rests on an assumption referred to in Chapter Two as the Kochen and Specker constraint. This is as follows:

(K and S) val(g(A)) = g(val(A))

The Kochen and Specker additivity requirement (referred to below as *Add*) follows from (K and S). Importantly, the Kochen and Specker 'no hidden variables' proof will not go through without *Add*.[14] In what follows I will be concerned to show that *Add* may not be quantum mechanically reasonable under certain circumstances and that quantum mechanics itself may point in the direction of some sort of contextual interpretation. This matter was originally broached by Albert et. al.[15]

In Chapter Two I discussed an argument due to Bell[16] which illustrated how the additivity requirement underlying theorems like those of Gleason[17] and Kochen and Specker could fail in contextual interpretations of quantum mechanics--interpretations of quantum mechanics sensitive to distinct measurement contexts. That argument will now be re-assessed in the light of the ABL-rule.

Consider a three-dimensional Hilbert space (since the Kochen and Specker argument applies to Hilbert spaces of three dimensions or more). Consider also a complete set of orthogonal (one-dimensional) projection operators $[\mathbf{P}_{(\phi_i)}]$ for this Hilbert space. This is a set of projection operators such that:

1) $\mathbf{P}_{(\phi_i)}\mathbf{P}_{(\phi_j)} = \mathbf{P}_{(\phi_j)}\mathbf{P}_{(\phi_i)} = \mathbf{P}_{(\phi_i)}\delta_{ij}$

2) $\Sigma_i \mathbf{P}_{(\phi_i)} = 1$

Since the eigenvalues of such operators are either 0 or 1, and because the operators add to unity, the additivity requirement, *Add*, on the measurement results means that upon measurement only one of the operators in the set $[\mathbf{P}_{(\phi_i)}]$ can take the value 1. The others must take the value 0.

This in turn has the following significance: if it is certain that measurement of $P_{(\emptyset_1)}$ on a system yields the value 0, and if it is certain that measurement of $P_{(\emptyset_2)}$ on that system at that time yields the value 0 ($\emptyset_1$ and $\emptyset_2$ orthogonal vectors in the three-dimensional Hilbert space), then on that system at that time measurement of $P_{(\psi)}$ is certain to yield the value 0 as well, where:

3) $\psi = c_1\emptyset_1 + c_2\emptyset_2$,

for arbitrary c_1, c_2. Thus if $P_{(\emptyset_3)}$ is orthogonal to $P_{(\emptyset_1)}$ and $P_{(\emptyset_2)}$, then in virtue of the fact that $P_{(\emptyset_1)} = 0$ and $P_{(\emptyset_2)} = 0$, $P_{(\emptyset_3)} = 1$.

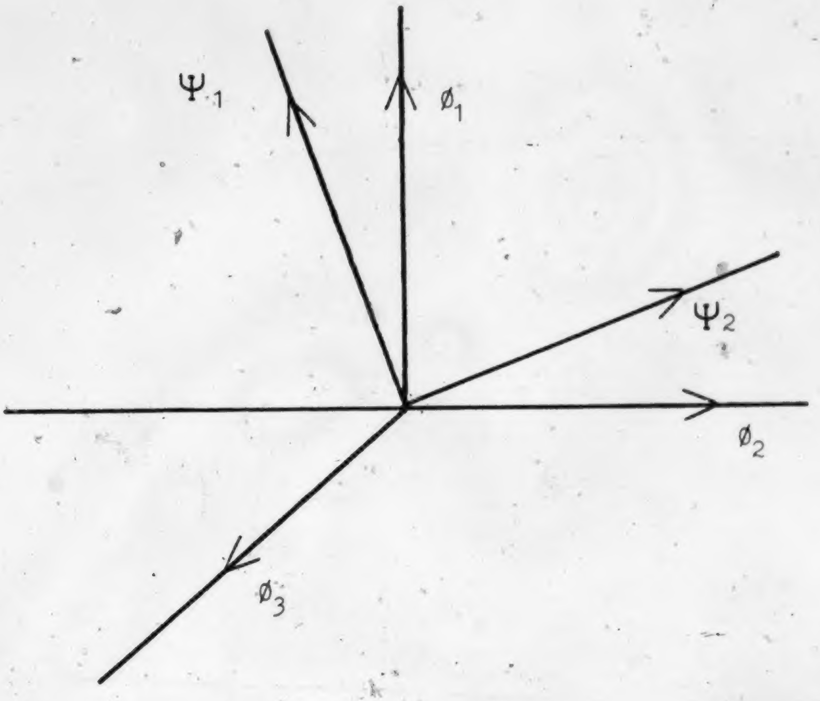
In Bell's discussion of the additivity requirement, the argument concerns two complete sets of orthogonal projection operators with a common member:

4) $P_{(\emptyset_1)}, P_{(\emptyset_2)}, P_{(\emptyset_3)} \quad ; \quad P_{(\psi_1)}, P_{(\psi_2)}, P_{(\emptyset_3)}$

Given the orthonormal basis $(\emptyset_1, \emptyset_2, \emptyset_3)$, one gets the other orthonormal basis $(\psi_1, \psi_2, \emptyset_3)$ by rotating the initial basis $d\theta$ about $\emptyset_3$. (See Figure 6-1). By the additivity requirement Add, if $P_{(\emptyset_3)} = 1$ relative to $(\emptyset_1, \emptyset_2, \emptyset_3)$ then $P_{(\emptyset_3)} = 1$ relative to $(\psi_1, \psi_2, \emptyset_3)$.

In Bell's original discussion of the requirement Add it was implicit that since $P_{(\emptyset_1)}$, $P_{(\emptyset_2)}$ and $P_{(\emptyset_3)}$ commute in pairs, (i.e., correspond to a maximal Boolean sub-algebra of the partial Boolean algebra of projection operators), they could be written as Borel functions of a maximal operator **A** as follows:

FIGURE 6-1 Two Orthonormal Bases for $V_3(C)$.



Ψ_1
$\emptyset_1$
Ψ_2
$\emptyset_2$
$\emptyset_3$

5) $P_{(\phi_1)} = g_1(\mathbf{A}),\ P_{(\phi_2)} = g_2(\mathbf{A}),\ P_{(\phi_3)} = g_3(\mathbf{A})$.

Similarly, since $P_{(\psi_1)}$, $P_{(\psi_2)}$ and $P_{(\phi_3)}$ commute in pairs:

6) $P_{(\psi_1)} = h_1(\mathbf{B}),\ P_{(\psi_2)} = h_2(\mathbf{B}),\ P_{(\phi_3)} = h_3(\mathbf{B})$

The maximal operators **A** and **B** do not commute with each other. Bell's original argument against Add was basically as follows: since the maximal operators **A** and **B** do not commute, their measurement will require different devices (or different configurations of the same device). Now one way to measure $P_{(\phi_3)}$ is to measure **A** and apply g_3 to the resultant value. Another way to measure $P_{(\phi_3)}$ is measure **B** and apply h_3 to the resultant value. Since **A** and **B** do not commute--and this apparently means that we must use different experimental arrangements to determine their values, Bell felt there was no a priori reason why $g_3(\mathbf{A}) = h_3(\mathbf{B})$. Bell concluded:

> The result of an observation may reasonably depend not only on the state of the system (including hidden variables) but also on the complete disposition of the apparatus.[18]

The idea here is this: in contextual hidden variables theories--those sensitive to the distinct ways a given observable can be measured--the requirement Add may simply not be reasonable. The situation is nicely summarized by Albert et al. as follows:

> Bell, some years ago, remarked that hidden variables theories could in principle be imagined which (by some averaging over the values of those variables) reproduce the statistical predictions of quantum mechanics, and which nonetheless (when some particular values of those variables are assumed) fail to satisfy that assumption; but the quantum mechanical statistics themselves, without any other such addenda, have always been thought to satisfy it . . .[19]

The assumption here is, of course, Add.

In the argument that follows (essentially due to Albert et al.), it will be claimed that in the interval between two measurements of the kind to which we apply the ABL-rule, the requirement Add will turn out to be unreasonable on quantum mechanical grounds. On the basis of this argument I will draw some conclusions concerning the contextuality of quantum mechanics.

Consider a three dimensional Hilbert space associated with a certain system S. Let the one-dimensional projection operators on the unit rays of that space be constants of the motion. Consider the arrangement of vectors given in Figure 6-2.

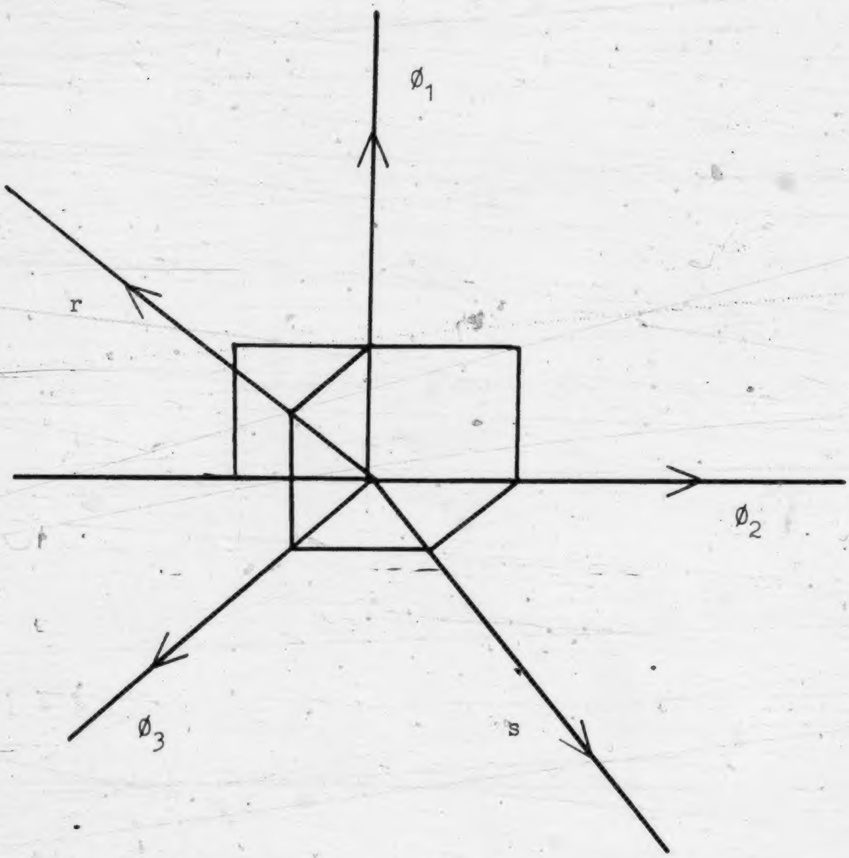
Suppose at time t_1 that the projection operator $P_{(r)}$ is measured and is found to have the value 1. Suppose at time t_2 that the projection operator $P_{(s)}$ is measured and is found to have the value 1. ($P_{(r)}$ and $P_{(s)}$ do not commute). What is the probability that measurement of the projection operator $P_{(\phi_i)}$ in the interval $t_1<t_n<t_2$ would have revealed the value 1. By the ABL-rule:

$$1)\quad P(P_{(\phi_i)}=1) = \frac{|\langle P_{(r)}=1 \mid P_{(\phi_i)}=1\rangle|^2 \, |\langle P_{(\phi_i)}=1 \mid P_{(s)}=1\rangle|^2}{\sum_i |\langle P_{(r)}=1 \mid P_{(\phi_i)}=i\rangle|^2 \, |\langle P_{(\phi_i)}=i \mid P_{(s)}=1\rangle|^2}$$

(Here the summation is over the possible eigenvalues i of $P_{(\phi_i)}$.) It follows trivially from equation (1) that in the interval $t_1<t_n<t_2$:

$$2)\quad P(P_{(r)} = 1) = P(P_{(s)} = 1) = 1$$

FIGURE 6-2 The Vectors r and s.


$\emptyset_1$
r
$\emptyset_2$
$\emptyset_3$
s

(Note: $\emptyset_1$, $\emptyset_2$, $\emptyset_3$ are mutually orthogonal. The vector r is orthogonal to $\emptyset_2$ and the vector s is orthogonal to $\emptyset_1$.)

Referring back to Figure 6-2, it can be seen that $\mathbf{P}_{(r)}$ is orthogonal to $\mathbf{P}_{(\emptyset_2)}$. Thus, in the interval $t_1<t_n<t_2$, $P(\mathbf{P}_{(\emptyset_2)} = 0) = 1$. Furthermore, the projection operator $\mathbf{P}_{(s)}$ is orthogonal to $\mathbf{P}_{(\emptyset_1)}$, so in the interval $t_1<t_n<t_2$ $P(\mathbf{P}_{(\emptyset_1)} = 0) = 1$. Since in this interval $P(\mathbf{P}_{(\emptyset_1)} = 0) = P(\mathbf{P}_{(\emptyset_2)} = 0) = 1$, then it must be the case that for this interval, $P(\mathbf{P}_{(\emptyset_3)} = 1) = 1$.

Consider now the three orthogonal projection operators $\mathbf{P}_{(\Psi_1)}$, $\mathbf{P}_{(\Psi_2)}$, $\mathbf{P}_{(\emptyset_3)}$, with Ψ_1, Ψ_2 and $\emptyset_3$ as illustrated in Figure 6-1. Imagine that the vectors r and s are as before. As before, $\mathbf{P}_{(r)} = 1$ at t_1 and $\mathbf{P}_{(s)} = 1$ at t_2. In this case, however, neither r nor s is orthogonal to either Ψ_1 or Ψ_2. So given $\mathbf{P}_{(r)} = 1$ at t_1 and $\mathbf{P}_{(s)} = 1$ at t_2 it does not follow that in the interval $t_1<t_n<t_2$, $P(\mathbf{P}_{(\Psi_1)} = 0) = P(\mathbf{P}_{(\Psi_2)} = 0) = 1$. Since this is the case, we can infer that in that interval, $P(\mathbf{P}_{(\emptyset_3)} = 1) \neq 1$. The requirement Add is thus violated. To see this, consider that:

3) $\Psi_2 = c_1\emptyset_1 + c_2\emptyset_2$,

for arbitrary c_1, c_2. From considerations based on the ABL-rule and given measurement results at t_1 and t_2 we have it that in the interval $t_1<t_n<t_2$, $P(\mathbf{P}_{(\emptyset_1)} = 0) = P(\mathbf{P}_{(\emptyset_2)} = 0) = 1$ and yet $P(\mathbf{P}_{(\Psi_2)} = 0) \neq 1$.

In particular, notice that the probabilities for measurement outcomes differ for $\mathbf{P}_{(\emptyset_3)}$, depending as to which orthonormal basis it is considered relative to. Relative to $(\emptyset_1, \emptyset_2, \emptyset_3)$, $P(\mathbf{P}_{(\emptyset_3)} = 1) = 1$. Relative to $(\Psi_1, \Psi_2, \emptyset_3)$, $P(\mathbf{P}_{(\emptyset_3)} = 1) \neq 1$. In Chapter Two of this study it was suggested that measurement contexts could be, from a theoretical point of view, associated with orthonormal bases in the appropriate Hilbert spaces. If this is right then we see how, on quantum mechanical grounds, probabilities for measurement outcomes vary from measurement context to measurement context.

One way to accommodate these distinct, orthonormal basis-relative, probabilities for measurement outcomes would be to associate with any projection operator $P_{(\emptyset_i)}$ a family of indempotent observables $[P_{(\emptyset_i)_c}]$, one observable for each orthonormal basis--i.e., measurement context c--to which the vector $\emptyset_i$ belongs.[20] Another way to proceed would be to introduce a multiplicity of probability measures, one for each orthonormal basis.[21] Thus, the probabilities for measurement outcomes would vary naturally from measurement context to measurement context. Either way, considerations of quantum mechanics with the ABL-rule suggest some form of contextual intepretation of quantum mechanics--at least in the interval between measurements.

This point concerning the contextuality suggested by quantum mechanics and the ABL-rule has been seen by Albert et al., who, in the light of the ABL-rule, comment on Bell's contextual suggestions for a hidden variables interpretation of quantum mechanics as follows:

> Bell has pointed out that in spite of the argument of Gleason and of Kochen and Specker, and without violating the statistical predictions of quantum mechanics, it can consistently be supposed (within certain hidden variables theories) that non-commuting observables can simultaneously be well-defined. The present considerations suggest something stronger: in spite of that argument and given those statistical predictions . . . it is inconsistent to assume anything else.[22]

SECTION SIX: EXTENSION TO THE BELL CASE

What now remains to be done is to examine the implications of the ABL-rule for the Bell argument and Bell- type experiments.

It was seen in Chapter Three that the concern of Bell's original argument was with the quantum spin correlation measurement statistics generated by spacelike separated pairs of spin-1/2 particles prepared in the singlet spin state. The particle-pairs are imagined to be prepared in the singlet spin state at a common source. They are then imagined to separate freely--so it is assumed that the particles are subject to no disturbing potentials while they are separating away from their common source. Measurements of selected spin components are then imagined to be performed on those particles using, for example, Stern-Gerlach magnets.

In particular, then, consideration is given to a pair of particles prepared into the singlet spin state Ψ_0 at t_1 (corresponding to the eigenvalue 0 for the total spin observable). At time t_2, $t_1<t_n<t_2$, we suppose that we measure spin along $\vec{a}$ on particle 1 in a pair 1+2. Quantum mechanics tells us that if we measure $S_1(\vec{a})$ we will find either +1 or -1. Assume that the value +1 is found. According to quantum mechanics, this measurement is simultaneously a measurement of spin along $\vec{a}$ on particle 2. In this case were we to measure $S_2(\vec{a})$ immediately after measuring $S_1(\vec{a})$, we would be sure to find the value -1.

The ABL-rule may be invoked for a consideration of probabilities for outcomes of measurements of spin components which we *could* have performed in the interval $t_1<t_n<t_2$. During this interval, the probability that we would find the value +1 for spin along $\vec{z}$ on particle 1, given that total spin S=0 at t_1 and that $S_1(\vec{a}) = +1$ at t_2, is given by:

$$1)\quad P(S_1(\vec{z}) = +1) = \frac{|\langle S=0|S_1(\vec{z})=+1\rangle|^2\,|\langle S_1(\vec{z})=+1|S_1(\vec{a})=+1\rangle|^2}{\Sigma_i|\langle S=0|S_1(\vec{z})=i\rangle|^2\,|\langle S_1(\vec{z})=i|S_1(\vec{a})=+1\rangle|^2}$$

(Here the summation is over the possible eigenvalues of $S_1(\vec{z})$). It follows from (1) that in the interval $t_1<t_n<t_2$:

$$2)\quad P(S=0) = P(S_1(\vec{a}) = +1) = 1$$

This means that had we measured either total spin or spin along $\vec{a}$ on particle 1 in the interval $t_1<t_n<t_2$, then we would certainly have found the values whose probabilities are given in (2). The two measurement results are thus determinate for the interval of the experiment. Of course, we didn't *know* until t_2 (when we measure $S_1(\vec{a})$ and find the value +1) that had we measured $S_1(\vec{a})$ in the interval of interest we would certainly have found $S_1(\vec{a}) = +1$. Measurement of $S_1(\vec{a})$ at t_2 thus removes our *ignorance* of what we would certainly have found had we measured $S_1(\vec{a})$ in the interval $t_1<t_n<t_2$.

The next stage of the argument requires some additional assumptions. By the principle of no retroactive causality,[23] the measurement result determinacy for $S_1(\vec{a})$ in the interval $t_1<t_n<t_2$ does not come about as the result of the measurement

performed at t_2. That is, measurement of any spin component at t_2 cannot affect the past--though it may well affect our knowledge of the past. In particular the $S_1(\overleftarrow{a})$ measurement result determinacy is not affected by measurements performed at t_2. Thus had we measured $S_1(\overleftarrow{b})$ rather than $S_1(\overleftarrow{a})$ at t_2, the $S_1(\overleftarrow{a})$ measurement result would still be determinate in the interval $t_1<t_n<t_2$.

By the principle of experimenter freedom,[24] at t_2 we could have measured any spin component--say, $S_1(\overleftarrow{b})$. Had we measured $S_1(\overleftarrow{b})$ rather than $S_1(\overleftarrow{a})$, quantum mechanics tells us that we would have found either +1 or -1. Suppose we found the value +1 for $S_1(\overleftarrow{b})$. By the ABL-rule we would conclude that in the interval $t_1<t_n<t_2$, $P(S_1(\overleftarrow{b}) = +1) = 1$. The $S_1(\overleftarrow{b})$ measurement result would thus be determinate in the interval of interest. That is, we have to find either +1 or -1 for $S_1(\overleftarrow{b})$ at t_2. Measurement of $S_1(\overleftarrow{b})$ at t_2 removes our ignorance of the value we would have found had we measured $S_1(\overleftarrow{b})$ in the interval $t_1<t_n<t_2$. Once again, by the principle of no retroactive causality, measurement of $S_1(\overleftarrow{b})$ at t_2 does not affect the $S_1(\overleftarrow{b})$ measurement result determinacy in the interval of interest. Now, by experimenter freedom, $S_1(\overleftarrow{b})$ can be any spin component one chooses. So if all that measurement of a spin component at t_2 does is to remove our ignorance of the value we would have found had we performed a measurement of that spin component in the interval (by the ABL-rule and the principle of no retroactive causality), then all spin component measurement results must be determinate in the interval of interest.

If all spin component measurement results are determinate in the interval $t_1<t_n<t_2$, then in that interval the set of spin components on a Bell-type pair-system has the property of classical determinacy of spin component measurement results. Thus for any particle pair 1+2 at time t_n, $t_1<t_n<t_2$, its hidden state $\lambda \in \Lambda$ will just be a list of those classically determinate measurement results--a list of values we would have found had we performed appropriate measurements. Since the $\lambda \in \Lambda$ employed by Bell (1964) can be understood as lists of measurement results in just this sense, then quantum mechanics plus the ABL-rule plus the principles of no retroactive causality and experimenter freedom implies the hidden variables assumption of the Bell argument.

In the Bell argument use is made of both the principle of experimenter freedom and the principle of no retroactive causality. The principle of experimenter freedom is used (as was seen in Chapter Five) to justify the claim that violations of Bell locality are violations of Einstein locality. The principle of no retroactive causality is implicit in the claim that fast analyzer-orientation changes cannot influence the distribution $\rho(\lambda)$ of the hidden variables $\lambda \in \Lambda$ at the source at the time of particle emission. Given the quantum prediction $A(\vec{a},\lambda) = -B(\vec{a},\lambda)$, Bell locality (and the principles of experimenter freedom and no retroactive causality), Bell concludes that there can be no hidden variables $\lambda \in \Lambda$.

In the argument above, the situation is reversed: Quantum mechanics, the ABL-rule and the principles of experimenter freedom and no retroactive causality imply hidden variables $\lambda \in \Lambda$ at least for the interval $t_1 < t_n < t_2$. Thus we conclude that quantum mechanics, under the above assumptions (all but one of which--the ABL-rule--Bell makes), cannot be Bell local (at least in the interval $t_1 < t_n < t_2$).

We thus have the following picture of what is going on in a Bell-type experiment: we start with an ensemble of systems (particle-pairs) each in the singlet spin state at t_1. Now measurement of spin along $\vec{a}$ on particle 1 at t_2 is usually taken, at t_2, to divide the initial ensemble into two sub-ensembles (one consisting of systems showing $S_1(\vec{a}) = +1$ the other consisting of systems showing $S_1(\vec{a}) = -1$). By the ABL-rule and the principle of no retroactive causality, however, it appears that each system in the initial ensemble belonged to one of these two sub-ensembles throughout the interval $t_1 < t_n < t_2$. (Measurement of t_2 reveals to us, for any system, which of the two sub-ensembles it belonged to).

Since, by experimenter freedom we can measure any component of spin--e.g., spin along $\vec{b}$--on particle 1 at t_2, it follows from the ABL-rule and the principle of no retroactive causality that for any direction $\vec{b}$, each system in the initial ensemble belongs to one of the two sub-ensembles associated for spin along $\vec{b}$. Thus, in the interval $t_1 < t_n < t_2$ spin component measurement results are classically determinate for every system in an ensemble of systems initially (at t_1) prepared in the singlet spin state and subsequently (at t_2) subjected to some spin component measurement. Since there is no distribution $\rho(\lambda)$ over these measurement results $\lambda \in \Lambda$ which will yield the quantum spin correlation measurement statistics, Bell locality must be violated.

The ABL-ensembles are precisely the ones referred to in discussions of Bell's theorem where systems are imagined to be prepared in the singlet state at t_1 and subsequently subjected to spin component measurements at t_2. The ABL-ensembles, given the arguments above, are ensembles of systems which have classically determinate spin component measurement results.

Contrary to widespread beliefs, *some* grounds exist for supposing that quantum mechanics is *not* Bell local. At least from the standpoint of quantum mechanics, it may not be an appropriate response to Bell's theorem to abandon the hidden variables assumption.

FOOTNOTES

[1] Aharonov et. al. (1964), p. 1411.

[2] See Clauser and Shimony (1978); van Fraassen (1982); Shimony (1984); Suppes (1984).

[3] Aharonov et. al. (1964).

[4] Albert et. al. (1985).

[5] See the discussion of the EPR argument in Chapter Three.

[6] See the discussion of the realist options at the beginning of Chapter Five.

[7] Aharonov et. al. (1964), p. 1414.

[8] The 'no hidden variables' proofs--and the Bell argument--concern systems at some time t. They are thus not dynamical arguments, though they may have consequences for certain dynamical arguments.

[9] Aharonov et al. (1964), p. 1412.

[10] Ibid., p. 1415.

[11] See Section Two above.

[12] Notice that what is determinate in the interval of interest are certain measurement results.

[13] Kochen and Specker (1967).

[14] See Bell (1966), (1982).

[15] Albert et. al. (1985). The present author had hoped to extend analysis to a treatment of the orthohelium atom in its lowest triplet state. Unexpected problems due to degeneracy put paid to this hope.

[16] See Bell (1966), (1982).

[17] Gleason (1957).

[18] Bell (1966), p. 451.

[19] Albert et. al. (1985), p. 6.

[20] As in HV-2. See Chapter Two.

[21] As in HV-5. See Chapter Two.

[22] Albert et. al. (1985), pp. 6-7.

[23] See Section Four of this Chapter.

[24] See Section Four of this Chapter.

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CHAPTER SEVEN

CONCLUSION

The three central questions addressed by this study are:

1) Is Bell's Theorem an inevitable consequence of the use of classical probability theory in the analysis of quantum spin correlation measurement statistics?

2) What is the relevance of Bell's theorem to the realist/anti-realist debate?

3) Is the standard view that quantum mechanics itself has no commitment to hidden variables correct?

The first question has been examined in connection with the work of Arthur Fine. Fine thinks that crucial to Bell's theorem is Bell's use of the random variable/phase space apparatus, and it is this setting for probabilistic analysis, rather than the hidden variables assumption, which should be abandoned in the light of the Bell result. It has been seen that Fine's three proposals for a non-standard and yet 'classical' analysis of the quantum spin correlation measurement statistics do not appear to be adequate.

While the derivation of Bell's theorem rests explicitly on use of the random variable/phase space apparatus, it has been seen in the course of the treatment of the second question that the appropriate response to Bell's theorem is not to give up either the random variable/phase space apparatus or the hidden variables assumption, but is rather to give up the constraint of Bell locality--and in particular the demand of Bell locality that the probability density ρ be a function of states $\lambda \in \Lambda$ alone. By taking $\rho = \rho(\vec{a},\vec{b},\lambda)$ we saw that it was *possible*, after all, to have an Einstein local, deterministic account of the quantum spin correlation statistics.

In my treatment of the second question, I examined the common lore that Bell's theorem rules out *local realism*. Various realist theses were examined and not all of them were found to be directly ruled out by Bell's theorem given the assumption of Bell locality. Some realist theories do not violate Bell locality because they do not make the hidden variables assumption (such theories were not immune to other problems, though). Furthermore it has been seen that Bell's theorem is relevant to the assessment of a number of anti-realist options. Finally, and as noted above, by making super-deterministic assumptions, we found that it was *possible* to sever the link between Einstein locality and Bell locality--so that an Einstein local hidden variables theory of the quantum spin correlation statistics could be produced.

Many quantum theorists, when faced with Bell's dilemma, have opted in favour of keeping Bell locality and of dropping the hidden variables assumption, thus regarding quantum mechanics as a Bell local theory. In my treatment of the third question I have argued that given specific details of the Bell case, quantum mechanics itself *may* require hidden variables in just the sense that is relevant to the Bell argument. If this is the case, then contrary to common lore, quantum mechanics cannot be Bell local.

In instrumentalist hands the common lore is seen as supporting the claim that quantum mechanics is to be assessed solely on the basis of its statistical predictions. When it comes to the task of explaining patterns of statistical results, the instrumentalist says, 'Don't ask!' Instrumentalism treats quantum mechanics as a rudderless ship of statistical techniques. In the context of the Bell argument, the appropriate response to instrumentalism is to shift the focus of the debate away from the hidden variables assumption and onto the Bell locality assumption.

The arguments advanced in my treatment of the third question suggest that quantum mechanics may itself make statistical predictions which are hard to reconcile with Bell locality. In the end, it *may* be Bell locality, rather than the hidden variables assumption, which has to go in the light of the Bell argument.

With an instrumentalist for its pilot, quantum mechanics becomes a rudderless ship. Via superdeterminism, the rudder may be restored--but at the expense of a pre-determined itinerary. As the poet Arthur Clough put it:

> And as of old from Sinai's top
> God said that God is One,
> By science strict so speaks He now
> To tell us there is none!
> Earth goes by chemic forces; Heaven's
> A méchanique Célèste!
> And heart and mind of human kind
> A watch-work as the rest.